I Choose the correct answer: (20 x 1 = 20)

1. The solution of \( \begin{vmatrix} 2x & 3 \\ 2 & 3 \end{vmatrix} \) is

   a) \( x = 1 \)     b) \( x = 2 \)     c) \( x = 3 \)     d) \( x = 0 \)

2. Given that the value of a third order determinant is 11 then the value of the determinant formed by the respective co-factors as its elements will be

   a) 11       b) 121       c) 1331       d) 0

3. The direction ratios of the vector \( \vec{a} = 5 \vec{i} + 7 \vec{j} - 6 \vec{k} \)

   a) \( (5, -7, 6) \)   b) \( (5, -7, -6) \)   c) \( (5, 7, -6) \)   d) \( (-5, -7, 6) \)

4. Sum of the squares of direction sines is

   a) 1       b) 0       c) -1       d) 2

5. If \( \frac{ax}{(x + 2)(2x - 3)} = \frac{2}{x + 2} + \frac{3}{2x - 3} \) then \( a = \)

   a) 4       b) 5       c) 7       d) 8

6. Sum of the binomial coefficients is

   a) \( 2^n \)       b) \( n^2 \)       c) \( 2^n \)       d) \( n + 17 \)

7. If \( a, b, c \) are in A.P. as well as in G.P. then

   a) \( a = b \neq c \)     b) \( a \neq b = c \)     c) \( a \neq b \neq c \)     d) \( a = b = c \)

8. If \( A, G, H \) are respectively arithmetic mean, geometric mean and harmonic mean then

   a) \( A > G > H \)     b) \( A < G > H \)     c) \( A < G < H \)     d) \( A > G < H \)

9. If two circles touch each other externally then the distance between their centres is

   a) \( r_1 - r_2 \)     b) \( \frac{r_1}{r_2} \)     c) \( \frac{r_2}{r_1} \)     d) \( r_1 + r_2 \)
10. Which of the following point lies in side the circle $x^2 + y^2 - 4x + 2y - 5 = 0$
   a) $(5, 10)$ b) $(-5, 7)$ c) $(9, 0)$ d) $(1, 1)$

11. The value of $\cos 135^0$ is ...
   a) $\sin 135^0$ b) $-\frac{1}{\sqrt{2}}$ c) $\frac{1}{\sqrt{2}}$ d) $\sin 45^0$

12. If the terminal side is collinear with the initial side in the opposite direction then the angle included is
   a) $0^\circ$ b) $90^\circ$ c) $180^\circ$ d) $270^\circ$

13. Which of the following is a function which is not one-to-one?
   a) $f : R \rightarrow R; f(x) = x^2$ b) $f : R \rightarrow R; f(x) = x^2 + 1$
   c) $f : R \rightarrow \{1,-1\}; f(x) = x - 1$ d) $f : R \rightarrow R; f(x) = -x$

14. Identify the correct statements
   (i) a constant function is a polynomial function
   (ii) a polynomial function is a quadratic function
   (iii) for linear function, inverse always exists
   (iv) A constant function is one-to-one only if the domain is a singleton set
   a) (i) and (iii) b) (i), (iii) and (iv) c) (ii) and (iii) d) (i) and (iv)

15. $\frac{1}{x} \log_5 e$ equal to
   a) $(\log_5 e)'$ b) $(\log_5 x)'$ c) $(\log_e 5)'$ d) $(\log_x 5)'$

16. If $y = 2x^3$ then $\frac{d^2 y}{dx^2}$
   a) $6x^2$ b) $12x$ c) $-12x$ d) $12$

17. $\int \frac{1}{\sqrt{3+4x}}dx = a) \frac{1}{2} \log \sqrt{3+4x} + c$ b) $\frac{1}{4} \log \sqrt{3+4x} + c$
   c) $2\log \sqrt{3+4x} + c$ d) $-\frac{1}{2} \log \sqrt{3+4x} + c$

18. $\int \sin^2 x dx = a) \frac{\sin^3 x}{3} + c$ b) $-\frac{\cos^2 x}{2} + c$
   c) $\frac{1}{2} \left[ x - \sin 2x \right] + c$ d) $\frac{1}{2} \left[ 1 + \sin 2x \right] + c$
19. Three coins are tossed. The probability of getting at least two heads is

a) \( \frac{3}{8} \)  

b) \( \frac{7}{8} \)  

c) \( \frac{1}{8} \)  

d) \( \frac{1}{2} \)

20. Two events A and B are independent, then \( P(A/B) = \)

a) \( P(A) \)  

b) \( P(A \cup B) \)  

c) \( P(A) = P(B) \)  

d) \( \frac{P(A)}{P(B)} \)

II Answer any 10 questions, question number 30 is compulsory  
\((10 \times 2 = 20)\)

21. Evaluate

\[
\begin{pmatrix}
ab & c(a+b) \\
bc & a(b+c) \\
ca & b(c+a)
\end{pmatrix}
\]

22. Find the sum of the vectors \( \vec{i} - \vec{j} + 2\vec{k} \)  

and \( 2\vec{i} + 3\vec{j} - 4\vec{k} \) and also the modulus of the sum.

23. Find the 7\(^{th}\) term of the sequence whose \( n^{th} \) term is \((-1)^{n+1} \left( \frac{n+1}{n} \right)\)

24. Is the point (7, -11) lie inside or outside the circle \( x^2 + y^2 - 10x = 0 \)?

25. Prove that \( \sin \theta \cos \theta \left\{ \sin \left( \frac{\pi}{2} - \theta \right) \cos \varepsilon c \theta + \cos \left( \frac{\pi}{2} - \theta \right) \sec \theta \right\} = 1 \)

26. Find the domain of the rational function \( f(x) = \frac{x^2 + x + 2}{x^3 - x} \)

27. Differentiate: \( 8e + 7 \tan x + x \frac{3}{2} \)

28. Integrate: \( \sin^5 x \)

29. An integer is chosen at random from the first fifty positive integers. What is the probability 

that the integer chosen is a prime or multiple of 4.

30. Define **Improper Fraction** of Rational Expression with one example

III Answer any 10 questions, question number 40 is compulsory  
\((10 \times 3 = 30)\)

31. Solve for \( x \) if \[
\begin{bmatrix}
1 & 1 & 2 \\
-1 & -4 & 1 \\
-1 & -1 & -2
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= [0]
\]
32. If ABC and \( A'B'C' \) are two triangles and G, \( G' \) be their corresponding centroids, prove that \( AA' + BB' + CC' = 3GG' \).

33. Resolve into partial fractions \( \frac{3x + 7}{x^2 - 3x + 2} \).

34. Find the sum of 101st term to 200th term of the series \( \sum_{n=1}^{\infty} \frac{1}{2^n} \).

35. Find the radius and center of the circle \((x-3)(x-5) + (y-7)(y-1) = 0\).

36. If \( \tan \alpha = \frac{1}{3} \) and \( \tan \beta = \frac{1}{7} \) show that \( 2\alpha + \beta = \frac{\pi}{4} \).

37. Solve the inequation: \( 7x^2 - 7x - 84 \geq 0 \).

38. Integrate: \( 3e^{7x} - 4\sec(4x + 3)\tan(4x + 3) + \frac{11}{x^3} \).

39. A card is drawn at random from a deck of 52 cards. What is the probability that the drawn card is (i) a queen or club card (ii) a queen or a black card.

40. Differentiate: \( \tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right) \).

IV Answer all the questions \((7 \times 5 = 35)\)

41. Prove that \( \begin{vmatrix} 1 + a & 1 & 1 \\ 1 & 1 + b & 1 \\ 1 & 1 & 1 + c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \) where a, b, c are non-zero real numbers and hence Evaluate the value of \( \begin{vmatrix} 1 + a & 1 & 1 \\ 1 & 1 + a & 1 \\ 1 & 1 & 1 + a \end{vmatrix} \).

(OR)

Prove that the medians of a triangle are concurrent.

42. Resolve into partial fractions \( \frac{x^2 - 2x - 9}{(x^2 + x + 6)(x + 1)} \).

(OR)

If a, b, c are in H.P., Prove that \( \frac{b + c}{b-a} + \frac{b+c}{b-c} = 2 \).
43. Find the circles which cuts orthogonally each of the following circles: \(x^2 + y^2 + 2x + 17y + 4 = 0,\) \(x^2 + y^2 + 7x + 6y + 11 = 0\) and \(x^2 + y^2 - x + 22y + 3 = 0\)

(OR)

Find \(k\) such that the equation \(12x^2 + 7xy - 12y^2 - x + 7y + k = 0\) represents a pair of straight lines. Find the separate equations of the straight lines and also the angle between them.

44. If \(\tan \theta + \sin \theta = p,\) \(\tan \theta - \sin \theta = q,\) and \(p > q\) then show that \(p^2 - q^2 = 4\sqrt{pq}\)

(OR)

If \(A + B + C = \pi,\) prove that \(\sin 2A - \sin 2B + \sin 2C = 4 \cos A \sin B \cos C\)

45. Differentiate \(\frac{\sin x + x \cos x}{x \sin x - \cos x}\) using quotient rule.

(OR)

Find \(\frac{dy}{dx}: x = 2 \cos \theta - \cos 2\theta,\) \(y = 2 \sin \theta - \sin 2\theta\)

46. Integrate \(\sqrt{x^2 + 3x + 10}\)

(OR)

Integrate: \((3x + 5)\sqrt{2x + 1}\)

47. If \(x\) is real, prove that the range of \(f(x) = \frac{x^2 - 2x + 4}{x^2 + 2x + 4}\) is between \(\left[\frac{1}{3}, 3\right]\)

(OR)

A husband and wife appear in an interview for two vacancies in the same post. The probability of husbands’ selection is \(\frac{1}{6}\) and that of wife’s selection is \(\frac{1}{5}\). What is the probability that (i) both of them will be selected (ii) only one of them will be selected (iii) none of them will be selected.

ALL THE BEST