CHAPTER 1

MATRICES AND DETERMINANTS

SINGULAR AND NON-SINGULAR MATRICES

- A square matrix A is said to be singular if | A | = 0
- A square matrix A is said to be non-singular matrix, if | A | ≠ 0

PROPERTIES OF DETERMINANTS

- The value of a determinant is unaltered by interchanging its rows and columns.
- If any two rows (columns) of a determinant are interchanged the determinant changes its sign but its numerical value is unaltered
- If two rows (columns) of a determinant are identical then the value of the determinant is zero.
- If every element in a row (or column) of a determinant is multiplied by a constant "k" then the value of the determinant is multiplied by k.
- If every element in any row (column) can be expressed as the sum of two quantities then given determinant can be expressed as the sum of two determinants of the same order with the elements of the remaining rows (columns) of both being the same.
- A determinant is unaltered when to each element of any row (column) is added to those of several other rows (columns) multiplied respectively by constant factors.
- Let A be any square matrix of order n then $|\boldsymbol{KA}| = \boldsymbol{K}^n|\boldsymbol{A}|$.

FACTOR METHOD

Application of Remainder theorem to determinants Theorem: If each element of a determinant (Δ) is polynomial in x and if Δ vanishes for x = a then (x − a) is a factor of Δ.

- This theorem is very much useful when we have to obtain the value of the determinant in 'factors' form. Thus, for example if on *putting* $a = b$ in the determinant Δ any two of its rows or columns become identical then Δ = 0 and hence by the above theorem a − b will be a factor of Δ.
- If r rows (column) are identical in a determinant of order n (n ≥ r) when we put $x = a$, then $(x - a)^{r-1}$ is a factor of Δ.
- (x + a) is a factor of the polynomial f(x) if and only if x = − a is a root of the equation f(x) = 0

m= {degree of diagonal} – {degree of factors}..

- **If** m= 0 then the other symmetric factor is a constant *(k).*
- **If** m = 1 then the other symmetric factor of degree 1 is *k(a + b + c)*
- If m= 2 then the other symmetric factor of degree 2 is $k(a^2 +$ *b2+c2+l(ab+bc+ca)*.

PRODUCT OF DETERMINANTS

- The determinant of the product matrix is equal to the product of the individual determinant values of the square matrices of same order.
- i.e. Let A and B be two square matrices of the same order. | AB | = | A | | B |

CHAPTER 3

ALGEBRA

PARTIAL FRACTIONS

1. TYPE OF METHODS

- linear factors none of which is repeated

$$\frac{x+a}{(x+b)(cx+d)} = \frac{A}{x+b} + \frac{B}{cx+d}$$

- linear factors some of which are repeated

$$\frac{x+a}{(x+b)(x+d)^2} = \frac{A}{x+b} + \frac{B}{x+d} + \frac{C}{(x+d)^2}$$

- quadratic factors , none of which is repeated

$$\frac{x}{(x+d)(x^2+b)} = \frac{A}{x+d} + \frac{Bx+C}{x^2+b}$$

2. PERMUTATIONS

- **Factorial** : the continued product of n natural numbers is called the n factorial and denoted by $n!$ $n! = 1 \times 2 \times 3 \times \ldots \ldots .. \times (n-1) \times n.$
- $0! = 1$
- **Fundamental principal of counting :** if the first job can be done in m ways , the second job in n ways , then the two jobs in succession can be completed in (m x n) ways.
- **Fundamental principal of addition:** if there are two jobs such that they can be performed independently in m and n ways respectively then either of two jobs can be performed in (m+n+) ways.
- **Permutation :** $nP_r = \frac{n!}{(n-r)!}$.
- **Permutation of objects not all distinct**

$$p + q = n \text{ is } \frac{n!}{p!\,q!}$$

- **Permutation when objects can repeat :** the number of permutation of n different things taken r at a time, when each may be repeated any number of times in each arrangements is n^r.

Circular permutations :

- The number of circular permutation of n distinct objects is $(n-1)!$
- **If** there are n things and if the direction is not taken into consideration the number of circular permutation is $\frac{1}{2}(n-1)!$

3. COMBINATIONS :

- $nC_r = \frac{n!}{(n-r)!r!}$
- $nC_n = 1$
- $nC_0 = 1$
- $nC_r = nC_{n-r}$
- $nC_r + nC_{r-1} = (n+1)C_r$
- $nC_x = nC_y \quad \rightarrow n = x + y$

4. MATHEMATICA INDUCTON :

Working Rule

- Put $n = 1$ then $p(1)$ is true
- Put $n = k$ then $p(k)$ is true
- Put $n = k + 1$ then $p(k+1)$ is true

5. BINOMIAL THEOREM :

- For any real number n, $(x+a)^n = nC_0x^na^0 + nC_1x^{n-1}a^1 + \cdots .. + nC_rx^{n-r}a^r$

6.GENERAL TERM :

- $\mathbf{T_{r+1}} = nC_rx^{n-r}a^r$
- $(x-a)^n = nC_0x^na^0 - nC_1x^{n-1}a^1 + \cdots .. + (-1)^n nC_rx^{n-r}a^r$
- $(x+1)^n = 1 + nC_1x^1 + nC_2x^2 + \cdots .. + nC_rx^r$
- $(1-x)^n = 1 - nC_1x^1 + nC_2x^2 - \cdots \ldots .. + (-1)^n nC_rx^r$

7. MIDDLE TERMS;

- When n is even , $\mathbf{T_{\frac{n}{2}+1}}$
- When n is odd $\mathbf{T_{\frac{n+1}{2}}}$ **and** $\mathbf{T_{\frac{n+3}{2}}}$

CHAPTER 4

SEQUENCE AND SERIES

ARITHMETIC PROGRESSION ;

- $t_n = a + (n-1)d$
- $d = t_2 - t_1$
- a is first term.
- $n = \frac{l-a}{d} + 1$

ARITHMETIC SERIES

- $S_n = \frac{n}{2}[2a + (n-1)d]$
- $S_n = \frac{n}{2}[a + l]$

GEOMETRIC PROGRESSION and SERIES

- $t_n = ar^{n-1}$
- $r = \frac{t_2}{t_1}$
- $S_n = \frac{a(r^n-1)}{r-1} \quad r > 1$
- $S_n = \frac{a(1-r^n)}{1-r} \quad r < 1$

HARMONIC PROGRESSION :

- $T_n = \frac{1}{a+(n-1)d}$

MEANS OF PROGRESSIONS

- **AM : if** A is the AM between a and b then $A = \frac{a+b}{2}$
- **GM :**if G is the geometric mean of the numbers of a and b is $G = \pm\sqrt{ab}$
- **HM : H** is called harmonic mean between a and b if $H = \frac{2ab}{a+b}$

BINOMIAL SERIES

- $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \ldots$
- $(1-x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \ldots$
- $(1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \cdots \ldots$
- $(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \cdots \ldots$

EXPONENTIAL SERIES

- $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \ldots$
- $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \ldots$

LOGARITMIC SERIES

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots ..$$

CHAPTER 5

ANALYTICAL GEOMETRY

1. What is locus?
   The path traced by a point when it moves according to specified geometrical conditions is called the locus of the point.
2. **EQUATIONS OF STRAIGHT LINES.**
   - Slope @ intercept form $y = mx + c$
   - Point slope form $y - y_1 = m(x - x_1)$
   - Two point form $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$
   - Intercept form $\frac{x}{a} + \frac{y}{b} = 1$
   - Normal form $x\cos\alpha + y\sin\alpha = p$
   - Parametric form $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$
   - General form $ax + by + c = 0$
3. Length of the perpendicular from (x_1, y_1) to the line $ax + by + c = 0$ is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$
4. Angle between the two lines $y = mx_1 + c_1$ and $y = mx_2 + c_2$ is $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$
5. **BASED TWO STRAIGHT LINES**
   - If the two straight lines are parallel then $m_1 = m_2$
   - If the two straight lines are perpendicular then $m_1 = -m_2$
   - Two parallel lines differ only by a constant.
6. Distance between two parallel line $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{C_1-C_2}{\sqrt{a^2+b^2}}\right|$
   - Any line parallel to $ax + by + c_1 = 0$ is of the form of $ax + by + k = 0$.

- Any line perpendicular to $ax + by + c_1 = 0$ is of the form $bx - ay + k = 0$ for some k.

7. Condition for three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ to be concurrent is $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

8. Equation of a line passing through the intersection of two given lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is

   $\boldsymbol{a_1x + b_1y + c_1 + \mu(a_2x + b_2y + c_2) = 0}$

9. Angle between the pair of straight lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is

   $\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

   - Product of slopes $m_1m_2 = \frac{a}{b}$
   - Sum of the slopes $m_1 + m_2 = \frac{-2h}{b}$
   - If the lines are parallel then $h^2 = ab$
   - If the straight lines are perpendicular then $a + b = 0$.

10. Condition for the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to represent a pair of straight lines is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.

11. Second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a circle if

   - $a = b$, {coeff. of x^2 = coeff. of y^2}
   - $h = 0$, no xy term

12. PARAMETRIC FORM

- $\boldsymbol{x = r\cos\theta, y = r\sin\theta}$ **are** called the parametric equation of circle $x^2 + y^2 = r^2$.
- Another parametric form $x = \frac{r(1-t^2)}{1+t^2}$ $\quad y = \frac{2rt}{1+t^2}$

13. CIRCLE

A Circle Is Locus of Point Which Moves In Such A Way That It's Distance From a fixed point is always constant.

14. EQUATION OF CIRCLE

- Centre (h, k) radius r $(x-h)^2 + (y-k)^2 = r^2$
- Diameter (x_1, y_1) and (x_2, y_2) given

  $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
- GE of Circle $x^2 + y^2 + 2gx + 2fy + c = 0$

  centre $(-g, -f)$, radius $= \sqrt{g^2 + f^2 - c}$

TANGENT

15. A tangent to a circle is a straight line which intersects the circle in exactly one point.

16. Equation of tangent at (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

   $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

17. The equation of tangent at (x_1, y_1) to the circle $x^2 + y^2 = a^2$ is $xx_1 + yy_1 = a^2$

18. The equation of any tangent to the circle is

   $y = mx + a\sqrt{1 + m^2}$

19. Length of the tangent from (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$PT = \sqrt{x_1{}^2 + y_1{}^2 + 2gx_1 + 2fy_1 + c}.$

- If $PT^2 = 0$ *then P lies on the circle.*
- If $PT^2 < 0$ *then P lies inside the circle.*
- If $PT^2 > 0$, *then P lies outside the circle.*

20. Condition for the line $y = mx + c$ to be tangent to the circle $x^2 + y^2 = a^2$ is $c^2 = a^2(1 + m^2)$.

21. The point of contact is $\left(\frac{-am}{\sqrt{1+m^2}}, \frac{a}{\sqrt{1+m^2}}\right)$

22. Equation of chord of contact of tangent from (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

23. CONCENTRIC CIRCLE

- Two or more circles having the same centre are called concentric circles.
- Two circles touch externally $c_1c_2 = r_1 + r_2$
- Two circles touch internally $c_1c_2 = r_1 - r_2$

24. ORTHOGONAL

Two circles are said to be orthogonal if the tangent at their point of intersection are at right angles.

25. Condition for two circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ is cut orthogonally $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

CHAPTER 6

TRIGONOMETRY FORMULAE

1. $\frac{180°}{\pi} = 1\ radian$

2. $1\ radian = 57°\ 16'$

3. $1° = 0.01746\ radian\ (app)$

4. $1° = \frac{\pi}{180}\ radian$

5.

$\sin\theta = \frac{1}{cosec\theta}$	$cosec\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\cot\theta = \frac{1}{\tan\theta}$	$\tan\theta = \frac{1}{\cot\theta}$

6. IDENTITY :

$sin^2\theta + cos^2\theta = 1$	$cos^2\theta = 1 - sin^2\theta$ $sin^2\theta = 1 - cos^2\theta$
$sec^2\theta - tan^2 = 1$	$sec^2\theta = 1 + tan^2$ $tan^2 = sec^2\theta - 1$
$cosec^2\theta - cot^2\theta = 1$	$cosec^2\theta = 1 + cot^2\theta$ $cot^2\theta = cosec^2\theta - 1$

7. COMPOUND ANGLES :

- $\cos(A + B) = \cos A \cos B - sinA\ sinB$
- $\cos(A - B) = \cos A \cos B + sinA\ sinB$
- $\sin(A + B) = sinA\ cosB + cosA\ sinB$
- $\sin(A - B) = sinA\ cosB - cosA\ sinB$

- $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
- $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
- $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

8. MULTIPLE ANGLE IDENTITIES

- $\sin 2A = 2\sin A\cos A$
- $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 1 - 2\sin^2 A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$
- $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

9. MULTIPLE ANGLE IDENTITIES WITH RATIO $\frac{A}{2}$

- $\sin A = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
- $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$
- $\cos A = \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
- $\cos A = 1 - 2\sin^2\frac{A}{2}$
- $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$
- $\sin^2\frac{A}{2} = \frac{1-\cos A}{2}$ or $1 - \cos A = 2\sin^2\frac{A}{2}$
- $\cos^2\frac{A}{2} = \frac{1+\cos A}{2}$ or $1 + \cos A = 2\cos^2\frac{A}{2}$
- $\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$
- $\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$
- $\tan^2\frac{A}{2} = \frac{1-\cos A}{1+\cos A}$

10. TRIGONOMETRICAL RATIOS INVOLVING 3A

- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$
- $\cot 3A = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$

11. TRANSFORMATION OF A PRODUCT INTO A SUM OR DIFFERENCE :

- $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
- $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
- $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
- $2\sin A\sin B = \cos(A-B) - \cos(A+B)$
- $-2\sin A\sin B = \cos(A+B) - \cos(A-B)$
- $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
- $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$
- $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$
- $\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

12. TRIGONOMETRICAL EQUATIONS

TYPE	SOLUTION
$\sin\theta = 0$	$\theta = n\pi \quad n \in Z$
$\cos\theta = 0$	$\theta = (2n+1)\frac{\pi}{2}$

$\tan\theta = 0$	$\theta = n\pi \quad n \epsilon Z$

13. GENERAL SOLUTION

TYPE	SOLUTION
$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n\alpha$
$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha$
$\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha$

14. PRINCIPAL VALUES LIES

- $\sin\theta = [-\frac{\pi}{2}, \frac{\pi}{2}]$
- $\cos\theta = [0, \pi]$
- $\tan\theta = [-\frac{\pi}{2}, \frac{\pi}{2}]$

15. NAPIER'S FORMULAE

- $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$
- $\tan\frac{B-A}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$
- $\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$

16. SINE FORMULAE

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

17. COSINE FORMULAE

- $a^2 = b^2 + c^2 - 2bc\cos A$
- $b^2 = c^2 + a^2 - 2ca\cos B$
- $c^2 = a^2 + b^2 - 2ab\cos C$

18. PROJECTION FORMULAE

- $a = b\cos C + c\cos B$
- $b = c\cos A + a\cos C$
- $c = a\cos B + b\cos A$

19. INVERSE TRIGONOMETRIC FUNCTION

- $tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$
- $tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)$
- $sin^{-1}x + sin^{-1}y = sin^{-1}[x\sqrt{1-y^2} +$

  $y\ 1-x2$

CHAPTER 8

DIFFERENTIAL CALCULUS

FUNDAMENTAL RESULTS ON LIMITS

- $\lim_{n\to c} kf(x) = k\lim_{n\to c} f(x)$

- $\lim_{n\to c} [f(x)+g(x)] = \lim_{n\to c} f(x) + \lim_{n\to c} g(x)$
- $\lim_{n\to c} [f(x)-g(x)] = \lim_{n\to c} f(x) - \lim_{n\to c} g(x)$
- $\lim_{n\to c} [f(x).g(x)] = \lim_{n\to c} f(x).\lim_{n\to c} g(x)$
- $\lim_{n\to c} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{n\to c} f(x)}{\lim_{n\to c} g(x)}$
- If $f(x) \leq g(x) then \lim_{n\to c} f(x) \leq \lim_{n\to c} g(x)$

SOME IMPORTANT RESULTS

- $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$
- $\lim_{n\to a} \frac{x^n-a^n}{x-a} = na^{n-1}$
- $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$
- $\lim_{x\to 0} \frac{e^x-1}{x} = 1$
- $\lim_{x\to 0} \frac{a^x-1}{x} = \log a$
- $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$
- $\lim_{x\to\infty} \left(1+\frac{k}{x}\right)^x = e^k$

[The following formulae differentiate with respect to x]

1. $x^n = nx^{n-1}$
2. $\sqrt{x} = \frac{1}{2\sqrt{x}}$
3. $\frac{1}{x^n} = -\frac{1}{x^{n+1}}$
4. $constant = 0$
5. $logx = \frac{1}{x}$
6. $\log_a x = \frac{1}{x}\log_a e$
7. $a^x = a^x \log a$
8. $e^x = e^x$
9. $e^{ax} = a\, e^{ax}$
10. $e^{-ax} = -a\, e^{ax}$
11. $sin\ x = cos\ x$
12. $cos\ x = -sin\ x$
13. $tan\ x = sec^2 x$
14. $cot\ x = -cosec^2\ x$
15. $sec\ x = sec\ x\ tan\ x$
16. $cosec\ x = -cosec\ x\ cot\ x$
17. $Sin\ ax = a\ cos\ ax$
18. $cos\ ax = -a\ sin\ ax$
19. ***product formula***
   - $\frac{d(uv)}{dx} = u.\frac{dv}{dx} + v\frac{du}{dx}$
   - $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx}-u.\frac{dv}{dx}}{v^2}$

INVERSE BASED FORMULAE

20. $sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
21. $cos^{-1} = \frac{-1}{\sqrt{1-x^2}}$
22. $tan^{-1}x = \frac{1}{1+x^2}$
23. $\cot^{-1} x = \frac{-1}{1+x^2}$

24. $sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

25. $cosec^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$

CHAPTER 9
INTEGRAL CALCULUS

1. $\int x^n \, dx = \frac{x^{n+1}}{n+1}$+c
2. $\int \frac{1}{x} \, dx = \log x + c$
3. $\int constant \; dx$

   $= \; constant \; with \; variable \; of \; integration$
4. $\int e^x \, dx = \frac{e^x}{1}$+c
5. $\int e^{ax} \, dx = \frac{e^{ax}}{a}$+c
6. $\int e^{-ax} \, dx = \frac{e^{-ax}}{-a}$+c
7. $\int a^x \, dx = \frac{a^x}{\log a}$+c
8. $\int (ae)^x \, dx = \frac{(ae)^x}{\log a + \log e} + c$
9. $\int sinx \; dx = -\cos x + c$
10. $\int sinax \; dx = -\frac{\cos ax}{a} + c$
11. $\int cosx \; dx = sinx + c$
12. $\int cosax \; dx = \frac{sinax}{a}$+c
13. $\int cosec^2 x \; dx = -\cot x + c$
14. $\int sec^2 x \; dx = \tan x + c$
15. $\int secx \, tanx \; dx = secx + c$
16. $\int cosecx \, cotx \; dx = -cosecx + c$
17. $\int (ax+b)^n \; dx = \frac{(ax+b)^n}{a\,(n+1)} + c$
18. $\int \frac{1}{(ax+b)} \; dx = \frac{\log\,(ax+b)}{a} + c$
19. $\int sin(ax+b) \; dx = -\frac{\cos(ax+b)}{a} + c$
20. $\int cos(ax+b) \; dx = \frac{\sin(ax+b)}{a} + c$
21. $\int cosec^2(ax+b) \; dx = -\frac{\cot(ax+b)}{a} + c$
22. $\int sec^2\,(ax+b) \; dx = \frac{\tan(ax+b)}{a} + c$
23. $\int sec(ax+b)\,tan(ax+b) \;\; dx = \frac{sec(ax+b)}{a}$
24. $\int cosec(ax+b)\,cot(ax+b) \;\; dx = -\frac{cose(ax+b)}{a}$
25. $\int \cot x \; dx = \log \sin x + c$
26. $\int \operatorname{cosec} x \; dx = \log \tan \frac{x}{2} + c$
27. $\int \tan x \; dx = \log \sec x + c$
28. $\frac{f'(x)}{f(x)} \; dx = \log f(x) + c$
29. $\int \frac{f'(x)}{\sqrt{f(x)}} \; dx = 2\sqrt{f(x)} + c$
30. $\int f'(x)\;[f(x)]^n = \frac{[f(x)]^{n+1}}{n+1} + c$
31. **PRODUCT IN INTEGRAL**
   - ❖ $\int u\,dv = uv - \int v\,du$
   - ✓ Let ' u' consider as ILATE method
   - ✓ I – Inverse function
   - ✓ L – logarithm
   - ✓ A – Algebraic expression
   - ✓ T – Trigonometric
   - ✓ E – Exponential
32. $\int e^{ax} \sin bx \; dx = \left(\frac{e^{ax}}{a^2+b^2}\right)(a \sin bx - bcosbx)$
33. $\int e^{ax} \cos\, bx \; dx = \left(\frac{e^{ax}}{a^2+b^2}\right)(a \cos bx + bsinbx)$
34. $\int \frac{1}{1+x^2} \; dx = tan^{-1}x + c$
35. $\int \frac{1}{\sqrt{1-x^2}} \; dx = sin^{-1}x + c$

36. $\int \frac{1}{1+(ax)^2}\,dx = \frac{1}{a}tan^{-1}(ax) + c$

37. $\int \frac{1}{\sqrt{1-(ax)^2}}\,dx = \frac{1}{a}sin^{-1}(ax) + c$

38. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log(\frac{a+x}{a-x}) + c$

39. $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log(\frac{x-a}{x+a}) + c$

40. $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}tan^{-1}\frac{x}{a} + c$

41. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = sin^{-1}\frac{x}{a} + c$

42. $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log(x + \sqrt{x^2-a^2}) + c$

43. $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log(x + \sqrt{x^2+a^2})$ +c

44. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}sin^{-1}\frac{x}{a} + c$

45. $\int \sqrt{x^2-a^2} = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x + \sqrt{x^2-a^2})$

46. $\int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log(x + \sqrt{x^2+a^2})$

CHAPTER 10
PROBABILITY

BASIC NOTES:

- $A \cup B$ stands for the occurrence of A (or) B
- $A \cap B$ stands for the simultaneous occurrence of A and B
- $\bar{A}$ *or* $\grave{A}$ *or* A^c stands for the non- occurrence of A
- $A \cap \bar{B}$ stands for the occurrence of only A.

PROBABILITY OF AN EVENT

- $P(A) = \frac{n(A)}{n(S)}$

AXIOMS OF PROBABILITY

- $0 \le p(A) \le 1$
- $p(S) = 1$
- If A and B are mutually exclusive events

  $P(A \cap B) = 0$: $P(A \cup B) = P(A) + P(B)$

SOME BASIC THEOREMS OF PROBABILITY

- $P(\emptyset) = 0$
- $P(\bar{A}) = 1 - P(A)$
- $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
- $P(\bar{A} \cap B) = P(B) - P(A \cap B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$
- $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

CONDITIONAL PROBABILITY

- $P(B/A) = \frac{P(A \cap B)}{P(A)}$
- $P(A/B) = \frac{P(A \cap B)}{P(B)}$

MULTIPLICATION THEOREM ON PROBABILITY

- $P(A \cap B) = P(A).P(B/A)$
- $P(A \cap B) = P(B).P(A/B)$

INDEPENDENT EVENTS

- If A and B are two independent events then

  $P(A \cap B) = P(A).P(B)$

- If $\bar{A}$ and $\bar{B}$ are two independent events then

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}).P(\bar{B})$$

BAYE'S THEOREM

- $P(A_i/B) = \frac{P(A).P(B/A_i)}{P(A_1).P(B/A_1)+P(A_2).P(B/A_2)+\cdots\ldots}$

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