CHAPTER 1
MATRICES AND DETERMINANTS

Define matrix:-

A matrix is a rectangular array of entries (or) elements put in rows and columns and within a square bracket (or) parenthesis

A matrix is always denoted by Capital letters and the elements are denoted by small letters

\[
\begin{pmatrix}
2 & 5 & 1 & 4 \\
6 & 3 & -2 & 0
\end{pmatrix}
\]

2 Rows
4 Columns

Dimensions : \((2 \times 4)\)

Order of a matrix

The order (or) size of a matrix is the number of rows and the number of columns that are present in a matrix

\[
A = \left( a_{ij} \right)_{m \times n}
\]
a\(_{ij}\) denotes the element present in the \(i^{th}\) row and \(j^{th}\) Column

\[
A = \begin{pmatrix}
\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23}
\end{array}
\end{pmatrix} = (A')^t
\]
i = 1,2 (rows)  j =  1, 2 ,3 (columns)

Determine the order of the matrix, and identify the specified elements.

\[
\begin{pmatrix}
1 & -1 \\
0 & 4 \\
2 & -1 \\
3 & 2
\end{pmatrix}
\]

Order 4 \times 2

\[
a_{22} = 4
\]

\[
a_{43} \text{ Doesn’t exist!}
\]

Types of matrices

1. Row matrix :
A matrix having only one row is called a row matrix (or) row vector

2. **Column matrix**
   A matrix having only one column is called column matrix (or) column vector
   \[
   \begin{bmatrix}
   6 \\
   9 \\
   -2
   \end{bmatrix}
   \quad \begin{bmatrix}
   6 \\
   -3 \\
   8 \\
   -9
   \end{bmatrix}
   \]
   3x1 4x1
   Any matrix of order 1x1 can be treated as either a row matrix (or) column matrix
   \[
   \begin{bmatrix}
   1 \\
   2 \\
   3 \\
   4
   \end{bmatrix}
   \]

3. **Square matrix:**
   A square matrix is a matrix in which the number of rows and the number of columns are equal
   \[
   X=\begin{pmatrix}
   -2 & -4 \\
   5 & 8
   \end{pmatrix}
   \quad Y=\begin{pmatrix}
   5 & 7 & -1 \\
   3 & -4 & 8 \\
   2 & -9 & 0
   \end{pmatrix}
   \]
   2 x 2 3 x 3
   \[
   \begin{bmatrix}
   2 & 3 & 4 \\
   8 & 9 & 2 \\
   -1 & 0 & -4
   \end{bmatrix}
   \]
   In general the number of elements in a square matrix of order “n” is \(n^2\)

4. **Diagonal matrix:**
   Diagonal matrix is a square matrix in which all the entries except the entries along the main diagonal are zero
   \[
   D=\begin{pmatrix}
   5 & 0 & 0 \\
   0 & 6 & 0 \\
   0 & 0 & 8
   \end{pmatrix}
   \]
   A square matrix in which all the entries off the main diagonal are zero. Example:
   \[
   \begin{pmatrix}
   2 & 0 & 0 & 0 \\
   0 & 1 & 0 & 0 \\
   0 & 1 & 0 & 0 \\
   0 & 0 & 0 & 8
   \end{pmatrix}
   \]
5. **Triangular matrix**

A square matrix in which all the entries above the main diagonal are zero

\[
T = \begin{pmatrix}
5 & 0 & 0 \\
8 & 6 & 0 \\
6 & -3 & 8
\end{pmatrix} \quad \text{(Lower triangular matrix)}
\]

A square matrix in which all the entries below the main diagonal are zero

\[
T = \begin{pmatrix}
5 & 4 & -7 \\
0 & 6 & -4 \\
0 & 0 & 8
\end{pmatrix} \quad \text{(Upper triangular matrix)}
\]

\[
\begin{bmatrix}
2 & 0 & 0 & 0 \\
3 & -1 & 0 & 0 \\
5 & 2 & 7 & 0 \\
4 & 0 & -2 & 8
\end{bmatrix}
\quad \begin{bmatrix}
8 & 2 & 5 & -3 \\
0 & 2 & 9 & 1 \\
0 & 0 & -4 & 2 \\
0 & 0 & 0 & 7
\end{bmatrix}
\]

6. **Scalar matrix:**

A scalar matrix is a diagonal matrix in which all the entries along the main diagonal are equal

\[
S = \begin{pmatrix}
5 & 0 & 0 \\
0 & 5 & 0 \\
0 & 0 & 5
\end{pmatrix}
\]

7. **Identity matrix (or) unit matrix**

An identity matrix (or) unit matrix is a scalar matrix in which entries along the main diagonal are equal to 1

\[
I_3 = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \quad I_2 = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\]

8. **Zero matrix(or) Null matrix**

A matrix said to be a zero matrix (or) null matrix if all the entries are zero

Zero matrix always denoted by \(0\)

\[
0 = \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix} \quad 0 = \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix} \quad 0 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

9. **Equality of matrices**

Two matrices \(A\) and \(B\) are said to be equal if

(i) they are of the same order
(ii) the corresponding elements of them are equal

10. **Transpose of a matrix**
The matrix obtained from the given matrix $A$ by interchanging its rows in to columns (or) its column in to rows is called transpose of $A$. It is denoted by $A^T$ (or) $A^t$.

If $A$ is order $2 \times 3$ then the order of $A^T$ is $3 \times 2$.

Example: Given that $B$ is of order $n \times m$. What is the order of $B$ transpose?

Given that $A$ is order $3 \times 4$. What is the order of $(A')^T = ?$

Note that $(A^T)^T = A$.

11. Multiplication of a matrix by a scalar:

Let $A$ be any matrix. Let $k$ be any non-zero scalar then the matrix $kA$ is obtained by multiplying all the entries of $A$ by the non-zero scalar $k$.

The order of the $A$ matrix is same as the order of $kA$.

12. Negative of a matrix:

Let $A$ be any matrix. The negative of a matrix $A$ is obtained by changing the sign of all the entries of $A$.

If $A = \begin{pmatrix} 4 & -7 \\ 5 & 0 \end{pmatrix}$ then $-A = \begin{pmatrix} -4 & 7 \\ -5 & 0 \end{pmatrix}$.

**Operation on matrices**

**Addition of two matrices**

Two matrices $A$ and $B$ can be added provided both the matrices of the same order and also the corresponding elements can be added.

Note that $A + (-A) = 0$.

Matrix addition is always commutative, i.e. $A + B = B + A$.

Matrix addition is always associative, i.e. $(A + B) + C = A + (B + C)$.

The additive identity of a matrix is a Zero matrix.

The additive inverse of $A$ is $-A$ (negative of $A$).

**Matrix multiplication**:

Two matrices $A$ and $B$ are said to be conformable for multiplication if the number of columns of the first matrix $A$ is equal to the number of rows of the second matrix $B$.

The procedure of multiplying is known as row–by-column multiplication.

If $A$ is of order $2 \times 3$ and $B$ is of order $3 \times 4$ then order of $AB$ is $2 \times 4$.

$A \times B = (\text{Number of rows of matrix } A) \times (\text{Number of columns of matrix } B)$.

Matrix multiplication is not commutative, i.e. $AB \neq BA$.

Matrix multiplication is always associative, i.e. $A(BC) = (AB)C$.

Matrix multiplication is distributive over addition, i.e. $A(B + C) = AB + AC$.
The product of any matrix with the unit matrix is the matrix itself i.e \( AI = IA = A \)

**DETERMINANTS**

What is the difference between matrix and determinant?

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Determinant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix has no value</td>
<td>Determinant has some value</td>
</tr>
<tr>
<td>Written with in parenthesis</td>
<td>Written with in two vertical lines</td>
</tr>
<tr>
<td>Number of rows need not be equal to Number of column</td>
<td>Number of rows = Number of columns</td>
</tr>
<tr>
<td>A constant ‘k’ is multiplied with all The elements of the given matrix</td>
<td>A constant ‘k’ can be multiplied with any one row (or) column</td>
</tr>
<tr>
<td>Matrix can be multiplied by using Row –by column method only</td>
<td>Determinant can be multiplied by using row by column (or) row by row (or) Column by column methods</td>
</tr>
</tbody>
</table>

The determinant is always a square form

**How to compute 2 x 2 determinant**

The value of \( \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc \) i.e \( \begin{vmatrix} 3 & 5 \\ -1 & 7 \end{vmatrix} = (21) - (-5) = 21+5=26 \)

**How to compute 3x3 determinant**

Example

\[
\begin{vmatrix}
1 & -2 & 3 \\
0 & 5 & 1 \\
3 & -2 & -6 \\
\end{vmatrix}
\]

\[
=1 \begin{vmatrix} 5 & 1 \\ -2 & -6 \end{vmatrix} - ( -2) \begin{vmatrix} 0 & 1 \\ 3 & -6 \end{vmatrix} +3 \begin{vmatrix} 0 & 5 \\ 3 & -2 \end{vmatrix} \\
= 1(-30+(-2)) +2 \{0-3\} +3 \{0-15\} \\
= 1(-30+2) +2(-3) +3(-15) \\
= -28 -6 -45 = -139
\]

**Singular and non Singular matrix**

A square matrix \( A \) is said to be singular if \( |A|=0 \)

A square matrix \( A \) is said to be non – singular if \( |A| \neq 0 \)
Minors

The minor determinant of \(|A| = |a_{ij}|\) is obtained by deleting the \(i^{th}\) row \(j^{th}\) column in which \(a_{ij}\) stands.

The minor of \((a_{ij})\) is denoted by \(M_{ij}\).

Cofactors

The cofactor is a signed minor.

Rule: \(A_{ij} = (-1)^{i+j} M_{ij}\)

\[
A = \begin{bmatrix}
1 & 2 & 3 \\
0 & 4 & 5 \\
1 & 0 & 6
\end{bmatrix} \quad \text{cofactor matrix} = \begin{bmatrix}
24 & 5 & -4 \\
-12 & 3 & 2 \\
-2 & -5 & 4
\end{bmatrix}
\]

\[
A_{11} = \begin{vmatrix}
2 & 3 \\
0 & 6
\end{vmatrix} = 24 \quad A_{12} = \begin{vmatrix}
1 & 5 \\
1 & 6
\end{vmatrix} = 5 \quad A_{13} = \begin{vmatrix}
0 & 4 \\
1 & 0
\end{vmatrix} = -4
\]

\[
A_{21} = \begin{vmatrix}
2 & 3 \\
0 & 6
\end{vmatrix} = -12 \quad A_{22} = \begin{vmatrix}
1 & 3 \\
1 & 6
\end{vmatrix} = 3 \quad A_{23} = \begin{vmatrix}
1 & 2 \\
1 & 0
\end{vmatrix} = 2
\]

\[
A_{31} = \begin{vmatrix}
2 & 3 \\
4 & 5
\end{vmatrix} = -2 \quad A_{32} = \begin{vmatrix}
1 & 3 \\
0 & 5
\end{vmatrix} = -5 \quad A_{33} = \begin{vmatrix}
1 & 2 \\
0 & 4
\end{vmatrix} = 4
\]

State factor theorem
If each element of a determinant (Δ) is a polynomial in x
For x = a  Δ vanishes then (x-a) is a factor of Δ

Note:

If r rows (column) are identical in a determinant of order n (n ≥ r) when we put x=a then (x-a)^r-1 is a factor of Δ

(x+a) is a factor of the polynomial f(x) if and only if x=-a is a root of the equation f(x)=0

Note
If ‘m’ = (the degree of the product of the factors)-(the degree of the product of the leading diagonal elements)

m = 0 then the other symmetric factor is a constant (k)
m = 1 then the other symmetric factor of degree is k (a+b+c)
m = 2 then the other symmetric factor of degree is 2 k (a^2 + b^2 + c^2) + l(ab + bc + ca)

Properties of Determinant:

1. The value of a determinant is unaltered by inter changing its rows and Columns - Prove it.

2. Prove that any two rows (columns) of a determinant are interchanged The determinant changes its sign but its numerical value is unaltered

3. Prove that If two rows (columns) of a determinant are identical then the value of the determinant is zero

4. Prove that every element in a row (or) column of a determinant is multiplied by a constant ‘k’ then the value of the determinant is multiplied by k

5. Prove that every element in any row (column) can be expressed as the sum of two quantities then given determinant can be expressed as the sum of two Determinant of the same order

6. Prove that the value of the determinant is unaltered when to each element of any row (column) is added to those of several other rows (column) multiplied respectively by constant factor

Note
Multiplying or dividing all entries of any one row (column) by the scalar is equivalent to multiplying or dividing the determinant by the same constant

PRODUCT OF DETERMINANTS

Note : |AB| = |A||B|

Two determinant can be multiplied by using (i) Row-by-Column method (ii) Row by Row method (iii)Column by Column method
Product of Determinants: The product of two determinants is expressed as:

\[
\begin{vmatrix}
  a_1 & b_1 & c_1 \\
  a_2 & b_2 & c_2 \\
  a_3 & b_3 & c_3
\end{vmatrix}
\times
\begin{vmatrix}
  a_1 & \beta_1 & \gamma_1 \\
  a_2 & \beta_2 & \gamma_2 \\
  a_3 & \beta_3 & \gamma_3
\end{vmatrix}
\]

\[
= \begin{vmatrix}
  a_1c_1 + b_1\beta_1 + c_1\gamma_1 & a_1c_2 + b_1\beta_2 + c_1\gamma_2 & a_1c_3 + b_1\beta_3 + c_1\gamma_3 \\
  a_2c_1 + b_2\beta_1 + c_2\gamma_1 & a_2c_2 + b_2\beta_2 + c_2\gamma_2 & a_2c_3 + b_2\beta_3 + c_2\gamma_3 \\
  a_3c_1 + b_3\beta_1 + c_3\gamma_1 & a_3c_2 + b_3\beta_2 + c_3\gamma_2 & a_3c_3 + b_3\beta_3 + c_3\gamma_3
\end{vmatrix}
\]

Where the rows have been multiplied by rows. We can also multiply the rows by columns.

RELATION BETWEEN A DETERMINANT AND ITS COFACTOR DETERMINANT

1. The sum of the products of the elements of any row of a determinant with the corresponding row of co factor determinant is equal to the value of the determinant
2. The sum of the product of the elements of any row of a determinant with any other row cofactor determinant is equal to 0

CHAPTER 2
VECTOR ALGEBRA

A quantity having only magnitude is called a scalar

Example  distance, area, volume, work done by force, real numbers

A quantity having both magnitude and direction is called a vector

Example  Displacement, force, velocity, acceleration, moment

Representation of a vector

A vector can be represented by a line segment

\[\vec{a} = \overrightarrow{AB} \quad \text{where } A \text{ is called initial point and } B \text{ is called terminal point}\]
Types of vectors

The magnitude of a vector \( \vec{OG} = \frac{\vec{a} + \vec{b} + \vec{c}}{3} \) (or) modulus of \( \vec{a} \) is the actual length

\[ |\vec{a}| = |\overrightarrow{AB}| = \overrightarrow{AB} \]

If the magnitude of a vector is 1 unit it is called unit vector.

If the magnitude of a vector is zero it is called zero vector.

Zero vector is also called Null vector Which is denoted by \( \vec{0} \).

For a zero vector the initial point is coincide with terminal point.

Equality of vectors

Unit vector always denoted by \( \hat{a} \). Where unit vector in the direction of \( a = \vec{a} \)

\[ a = \frac{\vec{a}}{|\vec{a}|} \]
Two vectors are said to be equal vector if they have the same magnitude and same direction.

A **vector quantity** has both magnitude and direction.

**Like vectors**

Vectors are said to be like they have the same direction.

**Unlike vectors**

Vectors are said to be unlike they have the opposite direction.

**Co-initial vectors**

Vectors having the same initial point.

**Co terminal vectors**

Vector having same end points.

**Collinear vectors**

Vectors are said to be collinear or parallel if they have same line of action.

**Coplanar vectors**

Three or more vectors lying in the same plane are called coplanar vectors.

**Negative of a vectors**

Two vectors are said to be negative to each other only if they have same magnitude and opposite in their direction.

\[ \overrightarrow{AB} = \overrightarrow{a}, \quad \overrightarrow{BA} = -\overrightarrow{a} \quad \text{i.e.} \overrightarrow{AB} = -\overrightarrow{BA} \]

**Operation on vectors**

Two vector can be added only if the end point of the first vector is equal to the starting point of the second vector.
\[ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \]

This law is known as ‘Triangle law of addition’ of vectors.

**Triangle Law:**
If two vectors are represented in magnitude and direction by two sides of a triangle taken in order, then their resultant (vector sum) is represented in magnitude and direction by the third side of the triangle.

**Parallelogram Law:**
If two vectors are represented in magnitude and direction by two adjacent sides of a parallelogram taken in order, then their resultant (vector sum) is represented in magnitude and direction by the diagonal passing through the point of intersection of the adjacent sides.

**Some basic rules:**
\[ \overrightarrow{AB} = -\overrightarrow{BA} \]
\[ \overrightarrow{a} + \overrightarrow{b} = \overrightarrow{b} + \overrightarrow{a} \]
\[ m(\overrightarrow{a} + \overrightarrow{b}) = m\overrightarrow{a} + m\overrightarrow{b} \]

**Linear combination of vectors**
If \( \overrightarrow{a} \) and \( \overrightarrow{b} \) are two vectors and \( x \) and \( y \) are real numbers, then the vector \( x\overrightarrow{a} + y\overrightarrow{b} \) is called the linear combination of \( \overrightarrow{a} \) and \( \overrightarrow{b} \).
Properties of addition of vectors

(i) Vector addition is always commutative

(ii) Vector addition is always associative

(iii) \( \vec{0} + \vec{a} = \vec{a} \) is called additive identity

(iv) \( \vec{a} + (-\vec{a}) = \vec{0} \) \(-\vec{a}\) is called additive inverse

Multiplication of a vector by a scalar

Let ‘k’ be a scalar. Then \( k\vec{a} \) is the scalar multiplication of \( \vec{a} \) by ‘k’

Note

If ‘k’ is negative then \( k\vec{a} \) is in the opposite direction

If ‘k’ is positive then \( k\vec{a} \) is in the same direction

If ‘k’ is Zero then \( k\vec{a} \) is a zero vector

If \( \vec{a} \) and \( \vec{b} \) are two vectors and \( m \) is a scalar then \( m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b} \)

Position vector

The starting point is same for all the points
\[ \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} \]
\[ \therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} \]

The position vector of a point that divides the line joining two given points whose position vectors are \( \vec{a} \) and \( \vec{b} \) internally in the given ratio \( m : n \) is \( \frac{m\vec{b} + n\vec{a}}{m+n} \)

(Externally in the given ratio \( m : n \) is \( \frac{m\vec{b} - n\vec{a}}{m-n} \))

If P is the mid point of AB then it divides AB in the ratio 1:1 then \( \overrightarrow{OP} = \frac{\vec{a} + \vec{b}}{2} \)

Condition for three point may be collinear \( (m+n)-m-n=0 \)
Conversely

If the scalars $x, y, z$ are such that $x + y + z = 0$ and $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$

Then $\vec{a}, \vec{b}$ and $\vec{c}$ are collinear

Note: The point of intersection of the medians of a triangle is the centroid of the triangle

Then the position vector of the centroid $\overrightarrow{OG} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$

Recall that the centroid divides any median in the ratio $2:1$

Thus,

$$\frac{DG}{GD} = \frac{1}{2}$$

Since $D$ is the mid-point of $BC$, we have $D = \left(\dfrac{\vec{b} + \vec{c}}{2}\right)$

Since $DG:GD = 2:1$, we can now determine $G$:

$$G = \frac{2 \times \dfrac{\vec{b} + \vec{c}}{2} + 1 \times \vec{a}}{2 + 1} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

If $G$ is the centroid of a triangle $ABC$, Prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$
1. Section formula: Internal division
\[ \overline{r} = \frac{m \overrightarrow{b} + n \overrightarrow{a}}{m + n} \]

2. Section formula: External division
\[ \overline{r} = \frac{m \overrightarrow{b} - n \overrightarrow{a}}{m - n} \]

3. Midpoint formula
\[ \overline{r} = \frac{\overrightarrow{a} + \overrightarrow{b}}{2} \]

4. Centroid of a triangle
\[ \overrightarrow{g} = \frac{\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}}{3} \]

5. Centroid of a tetrahedron
\[ \overrightarrow{g} = \frac{\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} + \overrightarrow{d}}{4} \]

6. Relation between direction cosines and direction ratios
\[ l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} \quad m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} \quad n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} \]
## Chapter 3
### Algebra

#### Permutation

**Factorial:**

The continued product of first $n$ natural numbers is called the “$n$ factorial:

(or) “factorial $n$”

<table>
<thead>
<tr>
<th>$1!$</th>
<th>$2! = 1 \times 2 = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3! = 1 \times 2 \times 3 = 6$</td>
<td>$4! = 1 \times 2 \times 3 \times 4 = 24$</td>
</tr>
<tr>
<td>$5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$</td>
<td>$6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Form of Rational Function</th>
<th>Form of the Partial Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{p(x) + q}{(x-a)(x-b)} ] $a \neq b$</td>
<td>[ \frac{A}{x-a} + \frac{B}{x-b} ]</td>
</tr>
<tr>
<td>[ \frac{p(x) + q}{(x-a)^2} ]</td>
<td>[ \frac{A}{x-a} + \frac{B}{(x-a)^2} ]</td>
</tr>
<tr>
<td>[ \frac{px^2 + qx + r}{(x-a)(x-b)(x-c)} ]</td>
<td>[ \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} ]</td>
</tr>
<tr>
<td>[ \frac{px^2 + qx + r}{(x-a)^2(x-b)} ]</td>
<td>[ \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b} ]</td>
</tr>
<tr>
<td>[ \frac{px^2 + qx + r}{(x-a)(x^2 + bx + c)} ]</td>
<td>[ \frac{A}{x-a} + \frac{Bx + C}{x^2 + bx + c} ]</td>
</tr>
</tbody>
</table>

Where, $x^2 + bx + c$ cannot be factorised further.

$A$, $B$, $C$ are real numbers that are to be determined.
\[ n! = 1 \times 2 \times 3 \times 4 \ldots \ldots (n-2) (n-1) n \]

\[(n-1)! = 1 \times 2 \times 3 \times 4 \ldots \ldots (n-2) (n-1)\]

It is denoted by \(n!\) or \(n\) 

**Factorial conversion**

\[6! = 5! \times 6 = 4! \times 5 \times 6\]

a. \(n! = (n-1)! n\)

b. \((n-1)! = (n-2)! (n-1)\)

c. \((n-r)! = (n-r-1)! (n-r)\)

d. \(r! = (r-1)! r\)

e. \((n-r+1)! = (n-r)! (n-r+1)\)

**Fundamental principle of counting**

1. **Multiplication principle**
2. **Addition principle**

**Fundamental principle of multiplication:**

If one operation can be performed in \(m\) ways and another operation can be performed in \(n\) ways then the two operations in succession can be performed in \(m \times n = m \times n\) ways.

**Fundamental principle of addition**

If there are two operations such that they can be performed independently in \(m\) ways and \(n\) ways respectively then either of the two operations can be performed in \((m+n)\) ways.

**CONCEPT OF PERMUTATION**

Permutation means “ARRANGEMENTS”

```
A
 /  \
B C
 /   \\
C B
```

```
B
 /  \
C A
 /   \\
A B
```

```
C
 /  \
A C
 /   \\
B A
```

```
ABC ACB BCA BAC CAB CBA
```

This type of permutation is called Linear Permutation.

**Permutations when objects can repeat:**

The number of permutation of $n$ different things, taken $r$ at a time, when each may be repeated any number of times in each arrangement is $n^r$.

Arrangements of things in a straight line is called **Linear Permutation**.

Arrangements of things around a circle are called **circular permutations**.

The circular permutation of $n$ different things is $(n-1)!$. 
CIRCULAR PERMUTATIONS

a. Number of circular arrangements (permutations) of n different things = (n−1)!

b. Number of circular arrangements (permutations) of n different things when clockwise and anticlockwise arrangement are not different i.e. when same observation can be made from both sides = \( \frac{1}{2} (n-1)! \).

c. Number of circular permutations of n different things, taken r at a time, when clockwise, and anticlockwise orders are taken as different, is = \( \frac{n!}{r!} \).

d. Number of circular permutations of n different things, taken r at a time, when clockwise and anticlockwise orders are not different, is = \( \frac{n!}{2r} \).

Note

When the clockwise and anticlockwise direction are indistinguishable the number of circular permutation of n things taken all at a time is \( \frac{(n-1)!}{2} \)

(i.e. garland with identical Pearls)

COMBINATION

A selection of any r things out of n things is called a combination of n things taken r at a time and it is denoted by \(^nC_r\) or \(^rC_n\) or \(C(n,r)\).

Difference between Permutation and Combination

\[ C(n,r) = \frac{n!}{r! (n-r)!} \]

\[ P(n,r) = \frac{n!}{(n-r)!} \]

- n = set size: the total number of items in the sample
- r = subset size: the number of items to be selected from the sample
1. In a Combination only selection is made

In permutation not only selection but also an arrangement in a definite order

2. Usually the number of permutation exceeds the number of combination

The formula for finding combinations \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \)

**CHAPTER 4**

**Sequence and series**

**Definition**
A sequence is an arrangement of numbers in a definite order according to some rule.

The sequence itself is denoted by \( \{a_n\} \)

There are two types of sequences

1. finite sequence (finite number of terms )  2 infinite sequence(infinite number of terms)
For each sequence there is an associated series which is obtained by replacing the comma in the ordered set of terms by plus symbols.

Some special types of sequences and their series

1. **Arithmetic progression**
   Arithmetic progression means each term except the first term is obtained by adding a fixed number to the immediately preceding term is called Arithmetic Progression.
   
   The \( n^{th} \) term of an A.P. \( t_n = a + (n-1)d \)

2. **Geometric progression**
   Geometric progression means each term except the first is obtained by multiplying the terms immediately preceding it by a fixed non zero number is called Geometric Progression.
   
   The \( n^{th} \) term of a G.P. \( t_n = ar^{n-1} \)

3. **Harmonic progression**
   A sequence of non zero numbers is said to be in Harmonic Progression if their reciprocals are in A.P.
   
   The \( n^{th} \) term of H.P. \( T_n = \frac{1}{n} = \frac{1}{a + (n-1)d} \)

**MEANS OF PROGRESSION**

1. **ARITHMETIC MEAN**
   Given two numbers \( a \) and \( b \) then \( x \) is called its arithmetic mean if \( a, x, b \) form an A.P.
   
   The arithmetic Mean (A.M) between \( a \) and \( b \) is given by \( x = \left( \frac{a + b}{2} \right) \)

2. **GEOMETRIC MEAN**
   Given two numbers \( a \) and \( b \), then \( x \) is called their geometric mean if \( a, x, b \) form a G.P.
   
   The Geometric Mean (G.M.) between \( a \) and \( b \) is given by \( x = \sqrt{ab} \)
3. **HORMONIC MEAN**

Given $a$ and $b$ then $x$ is called their harmonic mean (H.M.) if $a, x, b$ form a H.P.

That is $\frac{1}{a}, \frac{1}{x}, \frac{1}{c}$ form an A.P.

Then the H.M. between $a$ and $b$ is given by $x = \frac{2ab}{a+b}$
SOME SPECIAL TYPES OF SERIES

1. BINOMIAL SERIES:

\[(x + y)^0 = 1 \quad \text{0th row}\]

\[(x + y)^1 = 1x + 1y \quad \text{1st row}\]

\[(x + y)^2 = 1x^2 + 2xy + 1y^2 \quad \text{2nd row}\]

\[(x + y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3 \quad \text{3rd row}\]

\[(x + y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4 \quad \text{4th row}\]

\[(x + y)^5 = 1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5 \quad \text{5th row}\]

<table>
<thead>
<tr>
<th>Index</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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</tr>
<tr>
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<td>(\binom{1}{0}) ((=1)) (\binom{1}{1}) ((=1))</td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>4</td>
<td>(\binom{4}{0}) ((=1)) (\binom{4}{1}) ((=4)) (\binom{4}{2}) ((=6)) (\binom{4}{3}) ((=4)) (\binom{4}{4}) ((=1))</td>
</tr>
<tr>
<td>5</td>
<td>(\binom{5}{0}) ((=1)) (\binom{5}{1}) ((=5)) (\binom{5}{2}) ((=10)) (\binom{5}{3}) ((=10)) (\binom{5}{4}) ((=5)) (\binom{5}{5}) ((=1))</td>
</tr>
</tbody>
</table>

Pascal's triangle
When a binomial expression is raised to a power, the Binomial Theorem says that the co-efficients of the terms in the expansion of the binomial follow a distinct pattern.

For example:
\[(x+y)^0 = 1\]
\[(x+y)^1 = x + y\]
\[(x+y)^2 = x^2 + 2xy + y^2\]
\[(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3\]
\[(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4\]
\[(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5\]

The co-efficients can be shown as a triangle where each number is the sum of the two above it

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
```

This is often known as Pascal's Triangle, although Al-Kharaji, Yang Hui and Tartaglia had all produced similar results centuries earlier.

\[
(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \ldots \quad \text{provided } |x| < 1
\]

\[
(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots
\]

\[
(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots
\]

2. **EXPONENTIAL SERIES**

\[
e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots
\]

\[
e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots
\]

3. **LOGARITHMIC SERIES**

\[
\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \quad |x| < 1
\]

\[
\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \ldots
\]
LOCUS:-

The path traced by a point when it moves according to specified geometrical condition is called locus.

- Locus of a point equidistant from a given point in a plane is a circle.
• The locus of points equidistant from two given parallel lines is a line parallel to the two lines and midway between them.

• Locus of all points at a given distance from a given line is two straight lines parallel to the given line.
- The locus of points equidistant from two given intersecting lines is the bisectors of the angles formed by the lines.

- The locus of points equidistant from the sides of a given angle is the bisector of the angle.

### Different forms of straight lines

1. **Slope intercept form:**
   
   \[ y = mx + c \]
   
   Where \( m \) is the slope and \( c \) is the y intercept

### Find the Slope

A line passing through \((0, -2)\) and \((3, 4)\)

\[
\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{4 - (-2)}{3 - 0} = \frac{6}{3}
\]
2. **Point slope form**

   \[ y - y_1 = m(x - x_1) \]

   \( m = \text{slope} \)

   \( (x_1, y_1) = \text{any point on the line} \)

3. **Two point form**

   \[ \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \]

4. **Intercept form**

   \[ \frac{x}{a} + \frac{y}{b} = 1 \]

5. **Normal form**

   \[ (a, 0) \]
\[ \frac{x}{\sec \alpha} + \frac{y}{\cos \sec \alpha} = 1 \quad \text{(or)} \quad \frac{x \cos \alpha}{p} + \frac{y \sin \alpha}{p} = 1 \]

6. **Parametric form**
\[ \frac{x-x_1}{\cos \theta} = \frac{y-y_1}{\sin \theta} = r \]

Any point on the line can be taken as \((x_1 + r \cos \theta, y_1 + r \sin \theta)\)

General form of a straight line is \(ax + by + c = 0\)

7. **Perpendicular distance from a given point to a given line**
\[ \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \]

8. **Perpendicular distance from the origin to the line**
\[ \frac{c}{\sqrt{a^2 + b^2}} \]

9. **The slope of the line** \(ax + by + c = 0\) **is**
\[ \text{Slope} = \frac{C_0 \text{ coefficient of } x}{C_0 \text{ coefficient of } y} \]

**Angle between two straight lines**
\[ \theta = \tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \]
Distance between the two parallel straight lines \[ \text{D} = \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \]

Condition for the three straight lines to be concurrent

\[
\begin{vmatrix}
    a_1 & b_1 & c_1 \\
    a_2 & b_2 & c_2 \\
    a_3 & b_3 & c_3
\end{vmatrix} = 0
\]
PAIR OF STRAIGHT LINES

PAIR OF STRAIGHT LINE PASSING THROUGH THE ORIGIN

The general form of pair of straight line passing through the origin is \( ax^2 + 2hxy + by^2 = 0 \)

Note that

i) The straight lines are real and distinct if \( h^2 > ab \)
ii) The straight lines are coincident if \( h^2 = ab \)
iii) The straight lines are imaginary if \( h^2 < ab \)

Let \( y = m_1x \) and \( y = m_2x \) be the two straight lines passing through the origin

The sum of the slopes \( = m_1 + m_2 = \frac{-2h}{b} \)

The product of the slopes \( = m_1m_2 = \frac{a}{b} \)
1. General equation of a pair of straight lines passing through the origin
   \[ ax^2 + 2hxy + by^2 = 0 \]

2. General equation of a pair of straight lines
   \[ ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0. \]

3. Angle between a pair of straight lines passing through the origin
   Acute angle between lines
   \[ \tan \theta = \left| \frac{2(h^2 - ab)}{a + b} \right| \]

4. Condition for the equation \( ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \) to represent a pair of lines
   \[
   \begin{vmatrix}
   a & h & g \\
   h & b & f \\
   g & f & c
   \end{vmatrix} = 0
   \]

**ANGLE BETWEEN PAIR OF STRAIGHT LINES PASSING THROUGH THE ORIGIN**

Angle between the pair of straight lines \( = \theta = \tan^{-1} \left| \frac{2(h^2 - ab)}{a + b} \right| \)

**Note that**

If the straight lines are parallel, then \( h^2 = ab \)

If the straight lines are perpendicular then \( a + b = 0 \)

\( \text{Coeff of } x^2 + \text{Coeff of } y^2 = 0 \)

Condition for a general second degree equation represents a pair of straight lines Only if

\[
\Delta = abc + 2fgh - afh^2 - bg^2 - ch^2 = 0
\]

**CIRCLE**

**DEFINITION**

A circle is the locus of a point which moves in such a way that its distance from a fixed point is always constant. The fixed point is called the center of the circle and the constant distance is called the radius of the circle
EQUATION OF A CIRCLE WHEN THE CENTRE AND RADIUS ARE GIVEN

**Equation of a circle** in the centre radius form

\[(x - h)^2 + (y - k)^2 = r^2\]

where,

radius is 'r' and centre is \((h, k)\).

**Note that**

If the center of the circle is at the origin and radius \(r\) units, then the equation is \(x^2 + y^2 = r^2\)

THE EQUATION OF A CIRCLE IF THE END POINTS OF A DIAMETER ARE GIVEN

Equation of a circle in the diameter form

\[(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0\]

where,

\(A(x_1, y_1)\) and \(B(x_2, y_2)\) are the extremities of the diameter.

General equation of a circle

\[x^2 + y^2 + 2gx + 2fy + c = 0\]

where,

centre is \((-g, -f)\) and radius \(= \sqrt{g^2 + f^2 - c}\)

The condition for general equation of second degree, i.e., \(ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0\) represents a circle is

- \(a = b\), i.e., coefficient of \(x^2\) = coefficient of \(y^2\)
- \(h = 0\), i.e., coefficient of \(xy\) = 0.

**Parametric form of a circle**

\[x = a \cos \theta \quad , \quad y = a \sin \theta \quad \theta\ \text{is called the parameter and} \quad 0 \leq \theta \leq 2\pi\]
**Parametric** equations of a circle

\[ x = a \cos \theta, \quad y = a \sin \theta \] are the parametric equations of the circle

\[ x^2 + y^2 = a^2. \]

**Equation of tangent to a circle**

Equation of the tangent to a circle with origin as centre

The equation of the tangent to the circle \( x^2 + y^2 = a^2 \) at a point \( P(x_1, y_1) \) on it is given as \( xx_1 + yy_1 = a^2 \), where \( P(x_1, y_1) \) is a point in the circle and the tangent and 'a' is the radius.

**Equation of the tangent to a circle at a point \( (x_1, y_1) \)**

The equation of the tangent to the circle \( x^2 + y^2 + 2gx + 2fy + c = 0 \) at a point \( P(x_1, y_1) \) on it is \( xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0 \).

**Length of tangent to the circle from a point \( (x_1, y_1) \)**

\[ PT = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} \]

**Note:**

(i) If the point is on the circle then \( PT^2 = 0 \)

(ii) If the point is outside the circle then \( PT^2 > 0 \)

(iii) If the point is inside the circle then \( PT^2 < 0 \)

**Note**

(i) If the constant is zero then the circle passes through the origin

(ii) If the constant is positive the origin is outside the circle

(iii) If the constant is negative the origin is inside the circle

**Condition for the line \( y = mx + c \) to be a tangent to the circle \( x^2 + y^2 = a^2 \)**

The required condition is \( c^2 = a^2(1 + m^2) \)

The point of contact of the tangent is

\[ \left[ \frac{-am}{\sqrt{1 + m^2}}, \frac{a}{\sqrt{1 + m^2}} \right] \]

Number of tangents that can be drawn to the circle is \( = \) two
Circles touching internally and externally
If \( x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0 \) and
\( x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \) are the

equations of the two circles touching one another, then their centres are given by
\( C_1 \triangleq (-g_1, -f_1), C_2 \triangleq (-g_2, -f_2) \) and
their radii are
\[ r_1 = \sqrt{g_1^2 + f_1^2 - c_1}, \quad r_2 = \sqrt{g_2^2 + f_2^2 - c_2} \]

---

CHAPTER 6
TRIGONOMETRY

The word Trigonometry is derived from Greek word

Tri means Three Gono means Angles metry means measurement

Angles

If the rotation is counter clockwise, the angle is taken as POSITIVE
If the rotation is clockwise, the angle is taken as NEGATIVE
Measurement of angles

1 degree = 60 minutes = 60°
60 minutes = 60 seconds = 60°

Radian measure
A radian is the angle subtended at the center of a circle by an arc whose length is equal to the radius of the circle.

One Radian is written as $1^C$

![Diagram showing a radian](image)

\[ 2\pi r = \text{Circumference of a circle} \]

\[ \pi = \frac{\text{Circumference of a circle}}{2r} = (\text{diameter}) \]

\[ \pi = \frac{\text{Circumference of a circle}}{\text{Diameter of the circle}} \]

**Relation between Degrees and Radians**

The complete angle of a Circle $= 360 = 2\pi r$

If $r = 1$ Then $2\pi = 360$

\[ \pi = \frac{360}{2} = 180^0 \]

$\pi$ radians $= 180$ degrees

1 radian $= 57.3^0$

Radians $= \left( \frac{\pi}{180^0} \right) \times$ degrees

Degrees $= \left( \frac{180^0}{\pi} \right) \times$ radians
Radian

A unit for measuring angles, $180^\circ = \pi$ radians, and $360^\circ = 2\pi$ radians. The number of radians in an angle equals the number of radii it takes to measure a circular arc described by that angle.

Note: $360^\circ$ equals $2\pi$ radians because a complete circular arc has length equal to $2\pi$ times the radius.

Formula: $\theta = sr$

$\theta = \text{measure of the central angle}$ in radians
$s = \text{arc length}$
$r = \text{radius of the circle}$

Example:

$s = 10$
$r = 5$

$\theta = 10/5 = 2$ radians
Period of Trig Functions

Period=how long before it repeats- all 6 trigonometric functions are periodic or repeat. Sine and cosine functions have a period of $2\pi$, secant and cosecant functions have a period of $2\pi$, and tangent and cotangent have a period of $\pi$.

* Period of sine = 2 or 360°
* Period of cosine = 2 or 360°
* Period of tangent = $\pi$ or 180°
* Period of csc = same 2 or 360°
* Period of sec = 2 or 360°
* Period of cot = $\pi$ or 180°

Commonly used angles in degrees and radians

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
</tr>
<tr>
<td>30°</td>
<td>$\frac{\pi}{6}$</td>
</tr>
<tr>
<td>45°</td>
<td>$\frac{\pi}{4}$</td>
</tr>
<tr>
<td>60°</td>
<td>$\frac{\pi}{3}$</td>
</tr>
<tr>
<td>90°</td>
<td>$\frac{\pi}{2}$</td>
</tr>
<tr>
<td>120°</td>
<td>$\frac{2\pi}{3}$</td>
</tr>
<tr>
<td>135°</td>
<td>$\frac{3\pi}{4}$</td>
</tr>
<tr>
<td>150°</td>
<td>$\frac{5\pi}{6}$</td>
</tr>
<tr>
<td>180°</td>
<td>$\pi$</td>
</tr>
<tr>
<td>210°</td>
<td>$\frac{7\pi}{6}$</td>
</tr>
<tr>
<td>225°</td>
<td>$\frac{5\pi}{4}$</td>
</tr>
<tr>
<td>240°</td>
<td>$\frac{4\pi}{3}$</td>
</tr>
<tr>
<td>270°</td>
<td>$\frac{3\pi}{2}$</td>
</tr>
<tr>
<td>300°</td>
<td>$\frac{5\pi}{6}$</td>
</tr>
<tr>
<td>315°</td>
<td>$\frac{7\pi}{4}$</td>
</tr>
<tr>
<td>330°</td>
<td>$\frac{11\pi}{6}$</td>
</tr>
<tr>
<td>360°</td>
<td>$2\pi$</td>
</tr>
</tbody>
</table>
**Trigonometrical ratios**

If (2,3) is a point on the terminal side of \( \theta \), find all the six trigonometrical ratios.

If (-2,-3) is a point on the terminal side of \( \theta \) Find all the six trigonometrical ratios.

<table>
<thead>
<tr>
<th>Name</th>
<th>Ratio</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td>opposite/hypotenuse</td>
<td>Sin(( \theta ))</td>
</tr>
<tr>
<td>Cosine</td>
<td>adjacent/hypotenuse</td>
<td>Cos(( \theta ))</td>
</tr>
<tr>
<td>Tangent</td>
<td>opposite/adjacent</td>
<td>Tan(( \theta ))</td>
</tr>
<tr>
<td>Cosecant (1/Sine)</td>
<td>hypotenuse/opposite</td>
<td>Cosec(( \theta )) or csc(( \theta ))</td>
</tr>
<tr>
<td>Secant (1/Cosine)</td>
<td>hypotenuse/adjacent</td>
<td>Sec(( \theta ))</td>
</tr>
<tr>
<td>Cotangent (1/Tangent)</td>
<td>adjacent/opposite</td>
<td>Cot(( \theta ))</td>
</tr>
</tbody>
</table>

**Special properties of trigonometrical functions**

**Periodic function:**

A function is said to be a periodic function if for all values of \( x \)
\[ f(x+\alpha) = f(\alpha) \quad \alpha > 0 \] and \( \alpha \) is a constant. The least positive value of \( \alpha \) is called period of the function.

The Period of Sin \( x \), Cos \( x \), Cosec \( x \) and Sec \( x \) are \( 2\pi \)

The Period of tan \( x \) and Cot \( x \) have period \( \pi \)
ODD and EVEN function

If \( f(-x) = f(x) \), then the function is an even function.

Cos x, Sec x are all even functions.

If \( f(-x) = -f(x) \) then the function is an odd function.

Sin x, tan x, Cosec x, and Cot x are odd functions.

4 Sign Convention:

II quadrant
sin & cosec are +ve

I quadrant
All +ve

III quadrant
tan & cot are +ve

IV quadrant
cos & sec are +ve
(i) \( \sin(A+B) = \sin A \cos B + \cos A \sin B \)
(ii) \( \sin(A-B) = \sin A \cos B - \cos A \sin B \)
(iii) \( \cos(A+B) = \cos A \cos B - \sin A \sin B \)
(iv) \( \cos(A-B) = \cos A \cos B + \sin A \sin B \)
(v) \( \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \)
(vi) \( \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \)
(vii) \( \sin(A+B) \sin(A-B) = \sin^2 A - \sin^2 B \)
(viii) \( \cos(A+B) \cos(A-B) = \sin^2 A - \cos^2 B \)

**MULTIPLE ANGLES**

\( \sin 2A = 2 \sin A \cos A \)

\( \sin 2A = \frac{2 \tan A}{1 - \tan^2 A} \)

\( \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 \)

\( \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \)

\( \sin^2 A = \frac{1 - \cos 2A}{2} \)

\( \cos^2 A = \frac{1 + \cos 2A}{2} \)

\( \sin 3A = 3 \sin A - 4 \sin^3 A \)
Cos 3A = 4Cos^3 A - 3 Cos A

\[ \tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} \]

\[ \cot 3A = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1} \]

**SUB MULTIPLE ANGLES**

\[ \sin A = 2\sin \frac{A}{2} \cos \frac{A}{2} \]

\[ \cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} = 1 - 2\sin^2 \frac{A}{2} = 2\cos^2 \frac{A}{2} - 1 = \frac{1 - \tan^2 \frac{A}{2}}{1 + \tan^2 \frac{A}{2}} \]

\[ \sin^2 A = \frac{1 - \cos A}{2} \]

\[ \cos^2 A = \frac{1 + \cos A}{2} \]

\[ \tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}} \]

\[ \sin^2 A = \frac{1 - \cos A}{2} \]

\[ \cos^2 A = \frac{1 + \cos A}{2} \]

TRANSFORMATION OF A PRODUCT IN TO A SUM OR DIFFERENCE

\[ \sin (A+B) + \sin (A-B) = 2 \sin A \cos B \quad (1) \]

\[ \sin (A+B) - \sin (A-B) = 2 \cos A \sin B \quad (2) \]

\[ \cos (A+B) + \cos (A-B) = 2 \cos A \cos B \quad (3) \]

\[ \cos (A+B) - \cos (A-B) = -2 \sin A \sin B \quad (4) \]

\[ \cos (A-B) - \cos (A+B) = 2 \sin A \sin B \]

\[ \sin C + \sin D = 2 \sin \left(\frac{C+D}{2}\right) \cos \left(\frac{C-D}{2}\right) \quad (i) \]
\[
\sin C - \sin D = 2 \cos \left( \frac{C + D}{2} \right) \sin \left( \frac{C - D}{2} \right) \quad (ii)
\]
\[
\cos C + \cos D = 2 \cos \left( \frac{C + D}{2} \right) \cos \left( \frac{C - D}{2} \right) \quad (iii)
\]
\[
\cos C - \cos D = -2 \sin \left( \frac{C + D}{2} \right) \sin \left( \frac{C - D}{2} \right) \quad (iv)
\]

(or)
\[
\cos D - \cos C = 2 \sin \left( \frac{C + D}{2} \right) \sin \left( \frac{C - D}{2} \right)
\]

### TROGNOMETRICAL EQUATIONS

An equation involving trigonometric function is called a trigonometric equation

Note that: Trigonometric identities are different from trigonometric equations

Trigonometric equation is satisfied by only certain values

Where as the trigonometric identities are satisfied by for all values of \( \theta \)

Note: - (i) When we solve the equation of type \( \sin \theta = K \) or \( \tan \theta = K \), the principal value lies in between \( -\frac{\pi}{2}, \frac{\pi}{2} \) (or) \( [-\frac{\pi}{2}, \frac{\pi}{2}] \)

(ii) When we solve the equation of the type \( \cos \theta = K \), the principle value lies between \( 0 \) and \( \pi \) \( [0, \pi] \)

1. The general solution of \( \sin \theta = 0 \) and \( \tan \theta = 0 \) \( \Rightarrow \theta = n\pi \quad n \in \mathbb{Z} \)

2. The general solution of \( \cos \theta = 0 \) \( \Rightarrow \theta = (2n+1)\frac{\pi}{2} \quad n \in \mathbb{Z} \)

3. The general solution of \( \tan \theta = 0 \) \( \Rightarrow \theta = n\pi \)

4. The general solution of \( \sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n\alpha \)

5. The general solution of \( \cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha \)

6. The general solution of \( \tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha \)

7. Solving the equation of the form \( a\cos \theta + b\sin \theta = c \) (where \( c^2 \leq a^2 + b^2 \))
PROPERTIES OF TRIANGLES

1. **Sine formula**
   \[
   \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
   \]

2. **Derive Napier’s formula**
   \[
   \tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot\left(\frac{A}{2}\right)
   \]

3. **Cosine formula**
   \[a^2 = b^2 + c^2 - 2bc\cos A\]

4. **Projection formula**
   \[a = b\cos C + c\cos B\]

5. **Sub multiple angle formulae** (Half Angle formulae)
   (i) \(\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}\)
   (ii) \(\cos\left(\frac{A}{2}\right) = \sqrt{\frac{(s-a)}{bc}}\)
   (iii) \(\tan\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{(s-a)}}\)

6. **Area formula**
   \[
   \Delta = \frac{1}{2} ab \sin C
   \]

SOLUTION OF TRIANGLES

Given three sides \(a,b,c\) solve the triangle ABC (SSS type)

(Use the formula \(\tan\left(\frac{A}{2}\right) = \sqrt{\frac{(s-a)(s-c)}{s(s-a)}}\))

Given \(a=31\), \(b=42\), \(c=57\), find all the angles
Given \(a = 25\), \(b = 52\), \(c = 63\)

Suppose the numbers are smaller and hence we have to use Cosine formula

Given \(a = 8\), \(b = 9\), \(c = 10\) Find all the angles
Given \(a = 3\), \(b = 4\), and \(c = 6\)

Given two angles and any one side, to solve the triangle ABC (SAA-type)
(Use Sine formula)

Solve the triangle $\triangle ABC$ $A=35^\circ17'$; $C=45^\circ13'$; $b=42.1$

Given $a=1.53$, $B=85^\circ$, $C=70^\circ$

**Given two sides and the included angles (SAS type)**

Solve the triangle $\triangle ABC$ if $a=5$, $b=4$ and $C=68^\circ$

Solve the triangle $\triangle ABC$ if $a=54$, $b=42$ and $C=52^\circ6'$

**INVERSE TRIGONOMETRICAL FUNCTIONS**

$$
\sin^{-1}(\sin \alpha) = \alpha
\cos^{-1}(\cos \alpha) = \alpha
\tan^{-1}(\tan \alpha) = \alpha
\cot^{-1}(\cot \alpha) = \alpha
$$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}
\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}
\cot^{-1}\left(\frac{1}{x}\right) = \tan^{-1}x
$$

$$
\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)
\tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)
$$

**CHAPTER 7
FUNCTIONS AND GRAPHS**

**Constant**

A quantity which retains the same value throughout a mathematical process is called constant.

**Variable**

A variable is a quantity which can have different values.

**Intervals**

$\mathbb{R} = \text{REAL LINE}$

**i.e.,**

Different types of intervals

(i) Finite interval  (ii) infinite interval

Finite interval does not mean that the interval contains only a finite number of real numbers

Open interval :- If the interval contains neither of its end points
It is denoted by the parenthesis ( )
Example : (a , b )

Closed interval :- If the interval contains both of its end points
It is denoted by the square bracket [ ]
Example : [ a , b ]

Infinite intervals:

example (a,∞) (-∞, b) (-∞, ∞) [a, ∞) (-∞,a]

<table>
<thead>
<tr>
<th>interval notation</th>
<th>set notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>closed</td>
<td>[a, b]</td>
</tr>
<tr>
<td>open</td>
<td>(a, b)</td>
</tr>
<tr>
<td>half open (left)</td>
<td>(a, b]</td>
</tr>
<tr>
<td>half open (right)</td>
<td>[a, b)</td>
</tr>
<tr>
<td></td>
<td>(x : a &lt; x \leq b)</td>
</tr>
<tr>
<td></td>
<td>(x : a &lt; x &lt; b)</td>
</tr>
<tr>
<td></td>
<td>(x : a \leq x &lt; b)</td>
</tr>
</tbody>
</table>

Note that

We can’t write a closed interval by using ∞ or -∞
∞ and -∞ are not representatives of real numbers ∞, -∞ \(\not\in\) R

Neighborhood of a point:

Let a be any real number, Let \(\epsilon > 0\) be arbitrarily small real number then \((a-\epsilon,a+\epsilon)\) is called an \(\epsilon\) neighborhood of the point a

\[\bullet \quad \bullet\]

\[\text{closed interval} \quad \text{closed disk}\]

Types of variables:

(i) independent variable  (ii) dependent variable

Independent variable:
A variable is an independent variable when it has any arbitrary value

**Dependent variable:**

A variable is said to be dependent when its value depends on other variables.

**Example**

\[
\text{Area} = A = \pi r^2
\]

\[
\begin{aligned}
A & \text{ is dependent variable} \\
r & \text{ is independent variable}
\end{aligned}
\]

**Cartesian product:**

Let \( A = \{1, 2\} \) and \( B = \{a, b\} \) then \( A \times B \) is called Cartesian product of the two sets \( A \) and \( B \).

Note that \( A \times B \neq B \times A \) (in general).

**Relation**

Connect two sets by means of relation

\( R: A \longrightarrow B \) (read as \( A \) to \( B \))

Relation is a subset of Cartesian product \( A \times B \).

**FUNCTION:**

A function is a rule (or) law from a set \( A \) to \( B \) which assigns to each element of \( A \) a unique element of \( B \).

The set \( A \) is called domain of the function.

The set \( B \) is called co-domain of the function.

Which is denoted by \( f: A \longrightarrow B \).

\( f(A) \) is called the range of the function.

**Which is not a function?**

(i) Single element may not have two different element (or)

(ii) One element in the domain may not have not image in the co-domain
A function is a mathematical process that uniquely relates the value of one variable to the value of one (or more) other variables.

**Examples:**

- \( \sin(x) \)
- \( \exp(x) \)
- \( \cos(x) \)
- \( \ln(x) \)
- \( \tan(x) \)
- \( \tan^{-1}(x) \)
- \( x! \)
- \( x^3 + x^2 + 5x + 12 \)
- \( e^x \)
- \( \sqrt[2]{x} \)
- \( \cosh(x) \)
TYPES OF FUNCTION

ONE TO ONE FUNCTION

If a function relates any two distinct elements of its domain to two distinct elements of its range (or) co-domain, it is called a one to one function.

(or)

One – to – one function

A function \( f \) from \( A \) to \( B \) is called an one to one function if \( a, b \in A, a \neq b \) we must have \( f(a) \neq f(b) \) (distinct image)

(or)

A function is said to be one to one if each element of the range is associated with exactly one element of the domain

i.e., range of \( f \) is a subset of \( B \) \( \text{range} \subseteq \text{co-domain} \)

One to one is also called injective function

In to function:

A function \( f : A \rightarrow B \) is said to be into function if its range is a proper subset of its co-domain.

Range of \( f \) is a proper subset of \( B \)

i.e., range \( \subset \) co domain
On to function:

A function from $A$ to $B$ is said to be an on to function if its range is equal to its co-domain $B$

(or) A function $f$ is on to if to each element $b$ in the co-domain, there is at least one element $a$ in the domain such that $b = f(a)$

On to function is otherwise called an in to function

Range = co-domain

The other name for On to function is also surjective function

Many to one function

A function $f : A \rightarrow B$ is said to be many to one function if more than one element of $A$ is mapped in to the same element of $B$

In other words a function which is not one-to-one is called many to one and vice versa

Constant function

A function $f$ from $A$ to $B$ is called a constant function if every element of $A$ has the same image in $B$
Identity function:

A function \( f : A \rightarrow A \) is called an identity function if each element of \( A \) is associated with itself under \( f \).

In other words, \( f \) is an identity function of \( a \) if \( f(a) = a \) for all \( a \in A \).

\[
\begin{array}{c|c}
1 & 1 \\
6 & 6 \\
8 & 8 \\
\end{array}
\]

\( f : A \rightarrow A \) i.e., \( f(x) = x \)

Note that:

(i) Identity function is one to one and also onto function.
(ii) A function which is surjective and also injective is called bijective.
(iii) Identity function is a bijective function.
(iv) The identity function is the graph of the function \( (i.e,) y=x \) is a straight line passing through origin.

Note that:

1. A function is said to be injective if it is one-to-one.
2. A function is said to be bijective if it is both one to one and onto.

The Domain conversion

The following table illustrate the domain and range of certain functions.
<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^2$</td>
<td>$(-\infty, \infty)$</td>
<td>$[0, \infty)$</td>
</tr>
<tr>
<td>$y = \sqrt{x}$</td>
<td>$[0, \infty)$</td>
<td>$[0, \infty)$</td>
</tr>
<tr>
<td>$y = \frac{1}{x}$</td>
<td>$\mathbb{R} - {0}$ Non zero real numbers</td>
<td>$\mathbb{R} - {0}$</td>
</tr>
<tr>
<td>$y = \sqrt{1-x^2}$</td>
<td>$[-1, 1]$</td>
<td>$[0,1]$</td>
</tr>
<tr>
<td>$y = \sin x$</td>
<td>$(-\infty, \infty)$</td>
<td>$[-1, 1]$</td>
</tr>
<tr>
<td>$y = \cos x$</td>
<td>$(-\infty, \infty)$</td>
<td>$(-\infty, \infty)$</td>
</tr>
<tr>
<td>$y = \tan x$</td>
<td>$(\frac{-\pi}{2}, \frac{\pi}{2})$ principal solution</td>
<td>$(\frac{-\pi}{2}, \frac{\pi}{2})$ principal solution</td>
</tr>
<tr>
<td>$y = e^x$</td>
<td>$(-\infty, \infty)$</td>
<td>$(0,\infty)$</td>
</tr>
<tr>
<td>$y = \log x$</td>
<td>$(0, \infty)$</td>
<td>$(0, \infty)$</td>
</tr>
</tbody>
</table>

**Graph of a function:**

The graph of a function $f$ is a graph of the equation $y=f(x)$

A function have only one value of $f(x)$

**Note**

If we draw a vertical line to the above graph, it meets the curve at only one point

i.e., for every $x$ there is a unique $y$

**Vertical line Test**

The vertical line will intersect the graph of $f$ at the single point only

**INVERSE OF A FUNCTION**
Inverse of $f$ exists if and only if $f$ is one to one and also on to.

Suppose a function $f$ is only one to one function $f^{-1}$ is not exits.
And Suppose a function $f$ is only on to function then $f^{-1}$ is not exits.

Composition of function

Let $A$, $B$, and $C$ be any three sets and let $f: A \rightarrow B$ and $g: B \rightarrow C$.
Be any two function. Define a new function $(gof): A \rightarrow C$.

Note the domain of $g$ is the co domain of $f$.

The function $(gof)$ is called the composition of two functions $f$ and $g$. 
Composition of functions need not be commutative
  i.e., \( f \circ g \neq g \circ f \)

**Note that**

If \( f \circ g = g \circ f = I \) then \( g \) is called inverse of \( f \) or \( f \) is called inverse of \( g \)

\( f^{-1} \) exits then \( f \) is said to be invertible i.e., \( f \circ f^{-1} = f^{-1} \circ f = I \)

**Operation on functions**

\( f + g, f - g, \frac{f}{g} \) are also functions

**Note**

the product of two functions is different from composition two function

\( f + g \) and \( f g \) are different in meaning

**Linear function**

A function is said to be linear function if \( f(x) = mx + b \) where \( a \) and \( b \) are constants

![Linear Function](image)

\( f(x) = mx + b \)

**Note that**

(i) The graph of the linear function is a straight line
(ii) Inverse of a linear function always exists and also linear
(iii) A Polynomial function of degree 2 is called quadratic function

A function \( f(x) = ax^2 + bx + c \), Where \( a, b, c \in R\) and \( a \neq 0 \)

The graph of the function is always a parabola
(iv) A polynomial function of degree 3 is called cubic function
(v) A polynomial function of degree 4 is called bi quadratic function
(vi) A function \( \frac{f'(\infty)}{g(\infty)} \) is called a rational function (Provided \( g(x) \neq 0 \))

**Exponential functions:**

For any number \( a > 0, a \neq 1 \), the function \( f: R \rightarrow R \) defined by \( f(x) = a^x \) is called exponential function

The function \( f(x) = e^x \) whose curve lies between the corresponding to \( 2^x \) and \( 3^x \) ( "e" value lies between 2 and 3 )

Note that

(i) The inverse of the exponential function is a logarithmic function
(ii) The general form of logarithmic function is \( f(x) = \log_a x, \ x \neq 0 \)
(iii) The domain is \((0,\infty)\) of the logarithmic functions becomes the co domain of the exponential function and the co domain of the logarithmic function becomes the domain of the logarithmic exponential function

**RECIPROCAL OF A FUNCTION:**

A function is said to be reciprocal function if \( g(x) = \frac{1}{f(x)} \) \( f(x) \neq 0 \)
Note that
the graph of the \( y=1/x \) does not meet the either axes for finite real number
Which meets the \( x \) and \( y \) axes at infinity only

The inverse of the reciprocal function is the identity function

**ABSOLUTE VALUE FUNCTION (OR) MODULUS FUNCTION**

\[
f(x) = |x| = \begin{cases} 
  x, & \text{if } x \geq 0 \\
  -x, & \text{if } x < 0 
\end{cases}
\]

The domain is \( \mathbb{R} \) and co-domain is the set of all non negative real numbers

**STEP FUNCTIONS**

(1) GREATEST INTEGER FUNCTION
(2) LEAST INTEGER FUNCTION
(3) SIGNUM FUNCTION

**GREATEST INTEGER FUNCTION**

A function whose value at any real number \( x \) is the greatest integer less than or equal to \( x \) is called the greatest integer function

It is denoted by \( \lfloor x \rfloor \)

\[
[2.3] = 2 \quad [3.8] = 3 \quad [-2.1] = -3 \quad [0.5] = 0 \quad [-0.2] = -1 \quad [4] = 4
\]

Here the domain is \( \mathbb{R} \) where as the range is integers \( (\mathbb{Z}) \)
LEAST INTEGER FUNCTION:

A function whose value at any real number $x$ is the smallest integer greater than (or) equal to $x$ is called the least integer function.

It is denoted by $f(x) = \lfloor x \rfloor$

Note $\lfloor 2.5 \rfloor = 3$, $\lfloor 1.09 \rfloor = 2$, $\lfloor -2.9 \rfloor = -2$, $\lfloor 3 \rfloor = 3$

Here the domain is $\mathbb{R}$ whereas the range is integers ($\mathbb{Z}$)

SIGNUM FUNCTION

A function $f$ is defined by $f(x) = \begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$ is called Signum function.

Here the domain is $\mathbb{R}$ and the range is $\{-1, 0, 1\}$
Even function and odd function

(1) If $f(-x) = f(x)$ for all $x$ then the function is called even function

Example $f(x) = x^2$, $f(x) = x^2 + 2x^4$, $f(x) = \frac{1}{x^2}$, $f(x) = \cos x$

Note

(i) Many functions which are neither even nor odd
(ii) The even functions divides the y axis into two equal parts
(i.e., the graph of the even function is symmetrical about y-axis)

(2) If $f(x) = -f(-x)$ for all $x$ then the function is called odd function

Example $f(x) = x^3$, $f(x) = x - 2x^3$, $f(x) = \frac{1}{x}$, $f(x) = \sin x$

- The graph of the odd function is symmetrical about origin

TRIGONOMETRICAL FUNCTIONS

(1) Circular function

(2) Hyperbolic functions

QUADRATIC INEQUATIONS:

Let \( f(x) = ax^2 + bx + c \) \( f(x) \geq 0, f(x) \leq 0, f(x) > 0, f(x) < 0 \) are called quadratic in equations.

**CHAPTER 8**

DIFFERENTIAL CALCULUS

**Limit of a function**

**Left limit**

Let \( f(x) \) be a well defined function. Let us approach \( a \) through values less than \( a \). As \( x \) approaches in this way, the value of \( f(x) \) is called the left limit (if it exists).

It is written as \( \lim_{x \to a^-} f(x) = Lf(a) \).

**Right limit**

Let \( f(x) \) be a well defined function. Let us assume that \( a \) approaches \( a \) through values greater than \( a \). As \( x \) approaches \( a \) in this way, the value of \( f(x) \) in the limiting case is known as right limit (if it exists).

It is written as \( \lim_{x \to a^+} f(x) = Rf(a) \).

**Limit of a function at a point**
Let $f(x)$ be a well defined function of $x$. Let $a$ and $l$ be constants. As $x$ approaches $a$, let $f(x)$ approaches $l$ in the limiting case.

Then we say that the limit of $f(x)$ is $l$ as $x$ tends to $a$.

Thus we written as $\lim_{x \to a} f(x) = l$.

Note that

(i) If left limit = right limit we can say that limit of the function exists.
(ii) left limit $\neq$ right limit then the limit does not exits.
(iii) If any one the left or right does not exits then we can say the limit does not exits.

CONTINUITY OF A FUNCTION

Definition of a continuous function at a point $x=a$.

If $\lim_{x \to a} f(x)$ exists and is equal to $f(a)$ then the function $f(x)$ is said to be continuous function at $x=a$.

i.e.,

(i) $f(a)$ is defined
(ii) $\lim_{x \to a} f(x)$ exists
(iii) $\lim_{x \to a} f(x) = f(a)$

Discontinuous function at $x=a$.

(i) $f(a)$ is not defined
(ii) $\lim_{x \to a} f(x) = f(a)$ does not exist
(iii) $\lim_{x \to a} f(x) \neq f(a)$ therefore $f$ is discontinuous at $x=a$.
A function $f$ defined on a closed interval $[a, b]$ is said to be continuous at the end point $a$ if it is continuous from the right side at $a$, i.e.,

$$\lim_{{x \to a^+}} f(x) = f(a)$$

continuous at the end point $b$ if it is continuous from the left side at $b$, i.e.,

$$\lim_{{x \to b^-}} f(x) = f(b)$$

**Note**

(i) A function $f$ and $g$ are continuous functions at $x=c$, then the sum, difference, product and division of the functions are also continuous at $x=c$.

(ii) Every constant function is continuous.

(iii) The function $f(x)=x^n$, $x \in R$ is continuous.

(iv) Every polynomial function of degree $n$ is continuous.

Exponential function is continuous, i.e., $e^{f(x)}=a^x$

In particular

- the exponential function $f(x)=e^x$ is continuous
- Log function is continuous at all points of $R^+$
- Sin $x$ functions and cos $x$ functions all continuous at all points of $R$
Standard formulae for limits
1. $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

2. $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

3. $\lim_{x \to 0} \frac{a^x - 1}{x} = \log(a)$

4. $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

5. $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$

6. $\lim_{x \to 0} (1 + x)^{1/x} = e$

The following trigonometric formulae are very useful in this chapter:
1. $1 - \cos 2\theta = 2\sin^2 \theta$
2. $\cos C - \cos D = 2 \sin \frac{C + D}{2} \sin \frac{D - C}{2}$

**CHAPTER 8**

**DIFFERENTIATION**

List of formulae

<table>
<thead>
<tr>
<th>$y$</th>
<th>$\frac{dy}{dx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0</td>
</tr>
<tr>
<td>$x^n$</td>
<td>$n\ x^{n-1}$</td>
</tr>
<tr>
<td>$x$</td>
<td>1</td>
</tr>
<tr>
<td>$x^2$</td>
<td>$2x$</td>
</tr>
<tr>
<td>$x^3$</td>
<td>$3x^2$</td>
</tr>
<tr>
<td>$x^4$</td>
<td>$4x^3$</td>
</tr>
<tr>
<td>$\frac{1}{x}$</td>
<td>$-\frac{1}{x}$</td>
</tr>
<tr>
<td>$\frac{1}{x^2}$</td>
<td>$-\frac{2}{x^3}$</td>
</tr>
<tr>
<td>$\sqrt{x}$</td>
<td>$\frac{1}{2\sqrt{x}}$</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\frac{1}{\sqrt{x}} & \quad \frac{-1}{2x\sqrt{x}} \\
e^x & \quad e^x \\
e^{-x} & \quad -e^{-x} \\
\text{Log}x & \quad \frac{1}{x} \\
\sin x & \quad \cos x \\
\cos x & \quad -\sin x \\
\tan x & \quad \sec^2 x \\
\csc x & \quad -\csc x \cot x \\
\sec x & \quad \sec x \tan x \\
\cot x & \quad -\csc^2 x \\
\sec^2 x & \quad 2 \sec^2 x \tan x \\
\sin^{-1} x & \quad \frac{1}{\sqrt{1-x^2}} \\
\tan^{-1} x & \quad \frac{1}{1+x^2} \\
a^x & \quad a^x \log a
\end{align*}
\]

CHAPTER 9
INTEGRATION

Important formulae in integration

1. \[ \int kdx = kx + c \]
2. \( \int x \, dx = \frac{x^2}{2} + c \)

3. \( \int x^2 \, dx = \frac{x^3}{3} + c \)

4. \( \int \sqrt{x} \, dx = \frac{2}{3}x\sqrt{x} + c \)

5. \( \int \frac{1}{\sqrt{x}} \, dx = \frac{1}{2}x\sqrt{x} + c \)

6. \( \int \frac{1}{x} \, dx = \log|x| + c \)

7. \( \int \frac{1}{x^2} \, dx = -\frac{1}{x} + c \)

8. \( \int \frac{1}{x^3} \, dx = -\frac{2}{x^2} + c \)

9. \( \int e^x \, dx = e^x + c \)

10. \( \int e^{ax} \, dx = \frac{e^{ax}}{a} + c \)

11. \( \int e^{-x} \, dx = -e^{-x} + c \)

12. \( \int \log x \, dx = x \log x - x + c \)

13. \( \int xe^x \, dx = e^x(x - 1) + c \)

14. \( \int xe^{-x} \, dx = -e^{-x}(x + 1) + c \)

15. \( \int \sin x \, dx = -\cos x + c \)

16. \( \int \cos x \, dx = \sin x + c \)

17. \( \int \tan x \, dx = \log(\sec x) + c \)

18. \( \int \cot x \, dx = \log(\sin x) + c \)

19. \( \int \sec^2 x \, dx = \tan x + c \)

20. \( \int \sec x \tan x \, dx = \sec x + c \)
21. \[ \int \cos ec^2 x \, dx = - \cot x x + c \]
22. \[ \int \cos ecx \cot x \, dx = - \cos ecx + c \]
23. \[ \int \sec x \, dx = \log(\sec x + \tan x) + c \]
24. \[ \int \csc x \, dx = - \log(C \sec x + \cot x) + c \]
25. \[ \int \frac{1}{1 + x^2} \, dx = \tan^{-1} x + c \]
26. \[ \int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + c \]
27. \[ \int \frac{1}{\sqrt{1 - x^2}} \, dx = \sin^{-1} x + c \]
28. \[ \int \frac{-1}{\sqrt{1 - x^2}} \, dx = \cos^{-1} x + c \]
29. \[ \int a^x \, dx = \frac{a^x}{\log_e a} + c \]
30. \[ \int \frac{1}{\sqrt{1 + x^2}} \, dx = \log \left( x + \sqrt{1 + x^2} \right) + c \]
31. \[ \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + c \]
32. \[ \int \frac{1}{x^2 - a^2} \, dx = \log \left( \frac{x - a}{x + a} \right) + c \]
33. \[ \int \frac{1}{a^2 - x^2} \, dx = \log \left( \frac{a + x}{a - x} \right) + c \]
34. \[ \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} \left[ a \cos bx + b \sin bx \right] + c \]
35. \[ \int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} \left[ a \sin bx - b \cos bx \right] + c \]
36. \[ \int \frac{f'(x)}{f(x)} \, dx = \log(f(x)) + c \]
37. \[ \int \frac{f'(x)}{\sqrt{f(x)}} \, dx = 2\sqrt{f(x)} + c \]

**CHAPTER 10**

**PROBABILITY**

**Random experiment**: Random experiment is an experiment in which result cannot be predicted.

**TRIAL**: Performing a random experiment is called a trial.

**OUTCOME**: The result of the random experiment is called outcome.

**EVENT**: The set of single outcomes (or) combination of two or more outcomes is called event.

**SAMPLE SPACE**: The set of all possible outcomes of a random experiment is called sample space.
**IMPOSSIBLE OUTCOME:** The empty set is called impossible event

**SURE EVENT:** The whole sample space is called sure event

**MUTUALLY EXCLUSIVE EVENTS**

Two or more events are said to be mutually exclusive if they cannot occur simultaneously

$$S = \{HHH, THH, HTH, HHT, HTT, THT, TTH, TTT\}.$$  

Probability of an event \( E = P(E) = \frac{\text{Number of favourable outcome of an event}}{\text{Total number of all possible outcomes}} = \frac{n(E)}{n(S)} \)

\[ P(\emptyset) = 0 \quad \{ \text{Probability of an impossible event} = 0 \} \]

**AXIOMS ON PROBABILITY**

(i) \( P(A) \geq 0 \)

(ii) \( P(S) = 1 \quad \{ \text{Probability of a sure event} = 1 \} \)

(iii) If \( A_1, A_2, \ldots, A_n \) are mutually exclusive events then

\[ P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n) \]
ADDITION THEOREM ON PROBABILITY

If $A$ and $B$ are any two event such that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

CONDITIONAL PROBABILITY

A and $B$ are any event the conditional probability is defined as “the probability of event $B$ under the condition that the event $A$ has already occurred.

Conditional probability

If we need to find the probability of an event occurring given that another event has already occurred, then we are dealing with conditional probability.

If $A$ and $B$ are two events, then the conditional probability that $A$ occurs given that $B$ already has is written as $P(A | B)$ where:

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

or:

$$P(A | B) = \frac{P(A \text{ and } B)}{P(B)}$$

A frog climbing out of a well is affected by the weather. When it rains, he falls back down the well with a probability of $1/10$. In dry weather, he only falls back down with probability of $1/25$. The probability of rain is $1/5$.

Find the probability that given he falls it was a rainy day.
Let's start by drawing the tree diagram of these events:

We want:  
\[ P(\text{rains} \mid \text{falls}) = \frac{P(\text{rains and falls})}{P(\text{falls})} \]

From the tree we can see:  
\[ P(\text{rains and falls}) = \frac{1}{50} \]

and that  
\[ P(\text{falls}) = \frac{1}{50} + \frac{4}{125} \]

Therefore:  
\[ P(\text{rains} \mid \text{falls}) = \frac{\frac{1}{50}}{\frac{1}{50} + \frac{4}{125}} = \frac{5}{13} \]

Which is denoted by  
\[ P\left(\frac{B}{A}\right) \]

The formulae for calculating  
\[ P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}, \quad P(A) \geq o \]

\[ P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \geq o \]

**INDEPENDENT EVENTS:**
If the happening of an event A does not depend on the happening of event B they are called independent event
**Multiplication theorem on probability**

If A and B are independent events, then $P(A \cap B) = P(A)P(B)$

**Independent events** are events that have no impact on each other when they occur.

When two events A and B occur together, the combined event is expressed as A and B.

Moreover, when A and B are independent events, the probability of A and B is given as follows:

If A and B are independent events, $P(A \text{ and } B) = P(A) \times P(B)$.
UNIT TEST 1
VECTOR ALGEBRA
Part A

Answer all the questions

20 x 1 = 20

1. The position vector of A is \( \vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k} \), \( \vec{AB} = 5\hat{i} + 7\hat{j} + 6\hat{k} \) then the position vector of B is

(i) \( 7\hat{i} + 10\hat{j} + 10\hat{k} \)
(ii) \( 7\hat{i} - 10\hat{j} + 10\hat{k} \)
(iii) \( 7\hat{i} + 10\hat{j} - 10\hat{k} \)
(iv) \( -7\hat{i} + 10\hat{j} - 10\hat{k} \)

2. If \( \vec{a} \) is an non-zero vector and \( k \) is a scalar such that \( |k\vec{a}| = 1 \) then \( k = ? \)

(i) \( \vec{a} \)
(ii) \( 1 \)
(iii) \( \frac{1}{|a|} \)
(iv) \( \pm \frac{1}{|a|} \)

3. Let \( \vec{a} \vec{b} \vec{c} \) be the vector \( \overrightarrow{AB}, \overrightarrow{BC} \) determined by two adjacent sides of a regular hexagon ABCDEF. The vector represented by \( \overrightarrow{EF} \) is

(i) \( \vec{a} - \vec{b} \)
(ii) \( \vec{a} + \vec{b} \)
(iii) \( 2\vec{a} \)
(iv) \( -\vec{b} \)

4. If \( \vec{a} = 2\hat{i} + \hat{j} - 8\hat{k} \) and \( \vec{b} = \overrightarrow{OA} \) then the the magnitude of \( \vec{a} + \vec{b} = ? \)

(i) \( 13 \)
(ii) \( \frac{13}{3} \)
(iii) \( \frac{3}{13} \)
(iv) \( \frac{4}{13} \)

5. If the position vectors of P and Q are \( 2\hat{i} + 3\hat{j} + 7\hat{k}, 4\hat{i} - 3\hat{j} + 4\hat{k} \) then the direction cosines of \( \overrightarrow{PQ} \)

(i) \( \frac{2}{\sqrt{161}}, \frac{-6}{\sqrt{161}}, \frac{11}{\sqrt{161}} \)
(ii) \( \frac{-2}{\sqrt{161}}, \frac{-6}{\sqrt{161}}, \frac{-11}{\sqrt{161}} \)
(iii) \( 2, -1, 11 \)
(iv) \( 1, 2, 3 \)

6. If \( \vec{a} = \hat{i} + \hat{j} - 2\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}, \vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k} \) then a unit vector parallel to \( \vec{a} + \vec{b} + \vec{c} \) is

(i) \( \frac{-\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}} \)
(ii) \( \frac{\vec{i} - \hat{j} + \hat{k}}{\sqrt{3}} \)
(iii) \( \frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}} \)
(iv) \( \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} \)

7. Which of the following vectors has the same direction as the vector \( \hat{i} - 2\hat{j} \)

(i) \( -\hat{i} - 2\hat{j} \)
(ii) \( 2\hat{i} + 4\hat{j} \)
(iii) \( -3\hat{i} + 6\hat{j} \)
(iv) \( 3\hat{i} - 6\hat{j} \)

8. If the initial point of a vector \( -2\hat{i} - 3\hat{j} \) is \((-1, 5, 8) \) then the terminal point is

(i) \( 3\hat{i} + 2\hat{j} + 8\hat{k} \)
(ii) \( -3\hat{i} + 2\hat{j} + 8\hat{k} \)
(iii) \( -3\hat{i} - 2\hat{j} - 8\hat{k} \)
(iv) \( 3\hat{i} + 2\hat{j} - 8\hat{k} \)

9. If \( G \) is the centroid of a triangle \( ABC \) and \( G' \) is the centroid of a Triangle \( A'B'C' \) then \( \overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} \) is

(i) \( \frac{3\vec{G}}{1} \)
(ii) \( 3\vec{G} \)
(iii) \( 2\vec{G} \)
(iv) \( \frac{4\vec{G}}{1} \)
10. If $G$ is the centroid of a triangle $ABC$ then $\overline{GA} + \overline{GB} + \overline{GC}$ is equal to
   (i) $3(\overline{a} + \overline{b} + \overline{c})$  
   (ii) $\overline{OG}$  
   (iii) $\overline{0}$  
   (iv) $4\overline{OG}$

11. The position vector of $A$ and $B$ are $\overline{a}$ and $\overline{b}$, $P$ divides $AB$ in the ratio $3:1$.
    $Q$ is the mid point of $AP$. The position vector of $Q$ is
   (i) $\frac{5\overline{a} + 3\overline{b}}{8}$  
   (ii) $\frac{3\overline{a} + 5\overline{b}}{2}$  
   (iii) $\frac{5\overline{a} + 3\overline{b}}{4}$  
   (iv) $\frac{3\overline{a} + \overline{b}}{4}$

12. If $\overline{AB} = k\overline{AC}$ where $k$ is a scalar then
    (i) $A,B,C$ are collinear  
    (ii) $A,B,C$ are coplanar  
    (iii) $\overline{AB}, \overline{AC}$ have the same magnitude  
    (iv) $A,B,C$ are coincident

13. The direction cosines of the $2\overline{i} + \overline{j} - 2\overline{k}$ are
    (i) $\left(\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$  
    (ii) $\left(-\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$  
    (iii) $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$  
    (iv) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$

14. Given that $\overline{a} = \overline{i} + \overline{j} + 2\overline{k}$ and $\overline{b} = -2\overline{i} + 3\overline{j} + 5\overline{k}$ then $|\overline{a} + \overline{b}|$ is
   (i) $\sqrt{23}$  
   (ii) $\sqrt{50}$  
   (iii) $\sqrt{70}$  
   (iv) $\sqrt{66}$

15. Given $\overline{A}^2 = \overline{AB} + \overline{BC} + \overline{DA} + \overline{CD} = ?$
   (i) $\overline{DA}$  
   (ii) $\overline{CA}$  
   (iii) $\overline{0}$  
   (iv) $-\overline{AD}$

16. If $\overline{a} + 2\overline{b}$ and $3\overline{a} + m\overline{b}$ are parallel, then the value of $m$ is
   (i) 3  
   (ii) $\frac{1}{3}$  
   (iii) 6  
   (iv) $\frac{1}{6}$

17. If $\overline{a} + \overline{b} = 3\overline{i} - 4\overline{k}$, $\overline{a} - \overline{b} = \overline{i} + 2\overline{j}$ and $\overline{a} - \overline{b} = \overline{i} + 2\overline{j}$ then $\overline{a}$ is
   (i) $2\overline{i} - \overline{j}$  
   (ii) $2\overline{i} + \overline{j}$  
   (iii) $-2\overline{i} - \overline{j}$  
   (iv) $-2\overline{i} + \overline{j}$

18. $\overline{BA} = 3\overline{i} - 2\overline{j} + \overline{k}$ and the position vector of $B$ is $\overline{i} + 2\overline{j} - \overline{k}$ then the position vector of $A$ is
   (i) $3\overline{i} + 2\overline{j}$  
   (ii) $-3\overline{i} - 2\overline{j}$  
   (iii) $-6\overline{i} - 4\overline{j}$  
   (iv) $6\overline{i} - 4\overline{j}$

19. If the magnitude of the vector $\overline{i} + 2\overline{j} + 2\overline{k}$ is 3 units then the value of $l$ is
   (i) $\pm 3$  
   (ii) $\pm 2$  
   (iii) $\pm 1$  
   (iv) 0

20. The perimeter of the triangle formed by the vector $\overline{i} - 2\overline{j} + 2\overline{k}, \overline{i} - 2\overline{j} + 2\overline{k}$ and $2\overline{a}$ and $2\overline{b}$
   (i) 6  
   (ii) 7  
   (iii) 8  
   (iv) 9

Part B

Answer any 5 questions 5 x 6 = 30

21. (i) Find the magnitude and direction cosines of $2\overline{i} - \overline{j} + 7\overline{k}$
   (ii) Find the unit vector in the direction of $3\overline{i} + 4\overline{j} - 12\overline{k}$

22. The position vectors of the vertices $A, B, C$ of a triangle $ABC$ are respectively $2\overline{i} + 3\overline{j} + 4\overline{k}, -\overline{i} + 2\overline{j} - \overline{k}$ and $3\overline{i} - 5\overline{j} + 6\overline{k}$ Find the vectors determined by the sides and calculate the length of the sides.

23. Show that the points whose position vectors given by $-2\overline{i} + 3\overline{j} + 5\overline{k}, \overline{i} + 2\overline{j} + 3\overline{k}$ and $7\overline{i} - \overline{k}$ are collinear

24. Find the unit vector parallel to $3\overline{a} - 2\overline{b} + 4\overline{c}$ where $\overline{a} = 3\overline{i} - \overline{j} + 4\overline{k}, \overline{b} = -2\overline{i} + 4\overline{j} - 3\overline{k}; \overline{c} = \overline{i} + 2\overline{j} - \overline{k}$

25. The vertices of a triangle have position vectors $4\overline{i} + 5\overline{j} + 6\overline{k}, 5\overline{i} + 6\overline{j} + 4\overline{k}; 6\overline{i} + 4\overline{j} + 5\overline{k}$ Prove that the triangle is equilateral

26. Show that the vectors \(2\vec{i} - \vec{j} + \vec{k}, 3\vec{i} - 4\vec{j} - 4\vec{k}; \vec{i} - 3\vec{j} - 5\vec{k}\) form a right angled triangle.

27. If the vertices of a triangle have position vectors \(\vec{i} + 2\vec{j} + 3\vec{k}, 2\vec{i} + 3\vec{j} + \vec{k}\) and \(3\vec{i} + \vec{j} + 2\vec{k}\) Find the position vector of its centroid.

\[\text{Part C} \quad 5 \times 10 = 50\]

28. Show that the vectors \(5\vec{i} + 6\vec{j} + 7\vec{k}, 7\vec{i} - 8\vec{j} + 9\vec{k}, 3\vec{i} + 20\vec{j} + 5\vec{k}\) are coplanar.

29. Show that the points given by the vectors \(4\vec{i} + 5\vec{j} + \vec{k}, -\vec{j} - \vec{k}, 3\vec{i} + 9\vec{j} + 4\vec{k}\) and \(4\vec{i} + 4\vec{j} + 4\vec{k}\) are coplanar.

30. Examine whether the vectors \(\vec{i} + 3\vec{j} + \vec{k}; 2\vec{i} - \vec{j} - \vec{k}\) and \(7\vec{j} + 5\vec{k}\) are coplanar.

31. In a triangle ABC if D and E are the midpoints of sides AB and AC respectively Show that \(\overrightarrow{BE} + \overrightarrow{DE} = \frac{3}{2} \overrightarrow{BC}\).

32. If ABCD is a quadrilateral E and F are the mid-points of AC and BD respectively, prove that \(\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}\).

33. Prove that the sum of the vectors directed from the vertices to the mid points of opposite sides of a triangle is zero.

34. Prove that the line segment joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

35. By using vectors the mid points of two opposite sides of a quadrilateral and the mid points of the diagonals are the vertices of a parallelogram.

**UNIT TEST 2**

**MATRICES AND DETERMINANTS**

\[\text{Part A} \quad 1 \times 20 = 20\]

1. The order of matrix \(B = \begin{bmatrix} 1 & 2 & 5 & 7 \end{bmatrix}\) is
   (i) 1x4 (ii) 4 x1 (iii) 2x4 (iv) 1x1

2. Number of elements in a matrix of order 2x3 is
   (i) 5 (ii) 2 (iii) 3 (iv) 0

3. If \(A = \begin{bmatrix} 2 & 1 & 4 \\ -3 & 2 & 1 \end{bmatrix}\) and \(X + A = 0\) then matrix X is
   (i) \(\begin{bmatrix} 2 & 1 & 4 \\ -3 & 2 & 1 \end{bmatrix}\) (ii) \(\begin{bmatrix} -2 & -1 & -4 \\ 3 & -2 & -1 \end{bmatrix}\) (iii) \(\begin{bmatrix} -2 & -1 & -4 \\ 3 & -2 & 1 \end{bmatrix}\) (iv) \(\begin{bmatrix} 2 & 1 & 4 \\ 3 & -2 & -1 \end{bmatrix}\)

4. The product of the matrices \(\begin{bmatrix} 7 & 5 & 3 \\ 2 & 3 & 4 \end{bmatrix}\) is equal to
   (i) \([70]\) (ii) \([49]\) (iii) \([15]\) (iv) \(70\)
5. The type of the matrix \[
\begin{bmatrix}
\sqrt{2} & 0 & 0 \\
0 & \sqrt{3} & 0 \\
0 & 0 & \sqrt{3}
\end{bmatrix}
\]
is

(i) Scalar matrix  (ii) diagonal matrix  (iii) unit matrix  (iv) diagonal and scalar

6. If \[
\begin{bmatrix}
2 & x & -1 \\
0 & x & 3
\end{bmatrix} = [13]
\]
then the value of x is

(i) 5  (ii) 2  (iii) ± 3  (iv) ± 4

7. Matrix A is of order 2x3 and B is of order 3 x2 then order of matrix BA is

(i) 3x3  (ii) 2x3  (iii) 2x2  (iv) 3x2

8. If \[
\begin{bmatrix}
3 & -1 \\
2 & 5
\end{bmatrix} = \begin{bmatrix}
5 & 6 \\
1 & 7
\end{bmatrix}
\]
the order of matrix B is

(i) 3 x1  (ii) 1 x3  (iii) 3 x2  (iv) 1 x1

9. The true statements of the following are

(i) Every unit matrix is a scalar matrix but a scalar matrix need not be a unit matrix
(ii) Every scalar matrix is a diagonal matrix but a diagonal matrix need not be a scalar matrix
(iii) Every diagonal matrix is a square matrix but a square matrix need not be a diagonal matrix.

1. (i),(ii),(iii)  2. (i) and (ii)  3. (ii) and (iii)  4. (iii) and (i)

10. The matrix \[
\begin{bmatrix}
8 & 5 & 7 \\
0 & 6 & 4 \\
0 & 0 & 2
\end{bmatrix}
\]
is

(i) the upper triangular  (ii) lower triangular (iii) square  (iv) null

11. The minor of 2 in \[
\begin{vmatrix}
2 & -3 \\
6 & 0
\end{vmatrix}
\]
is

(i) 0  (ii) 1  (iii) 1  (iv) -3

12. The cofactor of -7 in \[
\begin{vmatrix}
2 & -3 & 5 \\
6 & 0 & 4 \\
1 & 5 & -7
\end{vmatrix}
\]
is

(i) -18  (ii) 18  (iii) -7  (iv) 7

13. If \[
A = \begin{bmatrix}
a_1 & b_1 & c_1 \\
a_2 & b_2 & c_2 \\
a_3 & b_3 & c_3
\end{bmatrix}
\]
and |A| = 2 then |3A| is

(i) 54  (ii) 6  (iii) 27  (iv) -54

14. In a third order determinant the cofactor of a_{23} is equal to the minor of a_{23} then the value of the minor is

(i) 1  (ii) Δ  (iii) -Δ  (iv) 0

15. The solution of \[
\begin{vmatrix}
2x & 3 \\
2 & 3
\end{vmatrix} = 0
\]
is

(i) x=1  (ii) x =2  (iii) x =3  (iv) x= 0
16. The value of \[
\begin{vmatrix}
1 & 1 & 1 \\
2x & 2y & 2z \\
3x & 3y & 3z
\end{vmatrix}
\]
is
(i) 1  (ii) \(xyz\)  (iii) \(x+y+z\)  (iv) 0

17. If \(\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}\) then \(\Delta = -\begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{vmatrix}\)
is equal to
(i) \(\Delta\)  (ii) \(-\Delta\)  (iii) \(3\Delta\)  (iv) \(-3\Delta\)

18. The value of the determinant \[
\begin{vmatrix} 1 & 2 & 3 \\ 7 & 6 & 5 \\ 1 & 2 & 3 \end{vmatrix}
\]
is
(i) 0  (ii) 5  (iii) 10  (iv) \(-10\)

19. If \(\Delta_1 = \begin{vmatrix} 1 & 4 & 3 \\ -1 & 1 & 5 \\ 3 & 2 & -1 \end{vmatrix}\) \(\Delta = 8\Delta_1\) and \(\Delta_2 = \begin{vmatrix} 2 & 8 & 6 \\ -2 & 2 & 10 \\ 6 & 4 & -2 \end{vmatrix}\) then
(i) \(\Delta_1 = 2\Delta\)  (ii) \(\Delta_1 = 4\Delta\)  (iii) \(\Delta_1 = 8\Delta\)  (iv) \(\Delta = 8\Delta_1\)

20. Two rows of a determinant \(\Delta\) are identical when \(x = -a\) then the factor of \(\Delta\) is
(i) \(x+a\)  (ii) \(x-a\)  (iii) \((x+a)^2\)  (iv) \((x-a)^2\)

**Part B**

**Answer any 5 question**  \(6 \times 5 = 30\)

21. Solve for \(x\) if \[
\begin{bmatrix} 1 & 1 & 2 \\ -1 & -4 & 1 \\ -1 & -1 & -2 \end{bmatrix}
\begin{bmatrix}
x \\
2 \\
1
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \\
0 \end{bmatrix}
\]

22. If \(A = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}\) Find \(k\) so that \(A^2 = Ka - 2I\)

23. Prove that \[
\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3
\]

24. Prove that \[
\begin{vmatrix} ab & -b^2 & bc \\ -ab & b^2 & -bc \\ ca & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2
\]

25. Prove that \[
\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ a-b & b-c & c-a \end{vmatrix} = a^3 + b^3 + c^3 - 3abc
\]

26. Using factor method, prove \[
\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)(x+9)
\]
27. Prove that  
\[
\begin{vmatrix}
1 & a & a^3 \\
1 & b & b^3 \\
1 & c & c^3
\end{vmatrix}
= (a-b)(b-c)(c-a)(a+b+c)
\]

28. (i) Prove that  
\[
\begin{vmatrix}
2x+y & x & y \\
2y+z & y & z \\
2z+x & z & x
\end{vmatrix}
= 0
\]
(ii) Evaluate  
\[
\begin{vmatrix}
x+2a & x+3a & x+4a \\
x+3a & x+4a & x+5a \\
x+4a & x+5a & x+6a
\end{vmatrix}
\]

29. Evaluate  
\[
\begin{vmatrix}
a-b & b-c & c-a \\
b-c & c-a & a-b \\
c-a & a-b & b-c
\end{vmatrix}
\]

30. Factorize  
\[
\begin{vmatrix}
a & b & c \\
b^2 & c^2 & a^2 \\
bc & ca & ab
\end{vmatrix}
\]

31. Prove that  
\[
\begin{vmatrix}
b^2 & (c+a)^2 & b^2 \\
c^2 & c^2 & (a+b)^2
\end{vmatrix}
= 2abc(a+b+c)^3
\]

32. Prove by factor method  
\[
\begin{vmatrix}
b+c & a-c & a-b \\
b-c & c+a & b-a \\
c-b & c-a & a+b
\end{vmatrix}
= 8abc
\]

33. Prove that  
\[
\begin{vmatrix}
a+\lambda & ab & ac \\
ab & b^2+\lambda & bc \\
ac & bc & c^2+\lambda
\end{vmatrix}
= \lambda^2(a^2+b^2+c^2+\lambda)
\]

34. Prove that  
\[
\begin{vmatrix}
1+a & 1 & 1 \\
1 & 1+b & 1 \\
1 & 1 & 1+c
\end{vmatrix}
= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)
\]
and hence
\[
\begin{vmatrix}
1+a & 1 & 1 \\
1 & 1+a & 1 \\
1 & 1 & 1+a
\end{vmatrix}
\]
find the value of  

35. If  \(x, y, z\) are all different and  
\[
\begin{vmatrix}
x & x^2 & 1-x^3 \\
y & y^2 & 1-y^3 \\
z & z^2 & 1-z^3
\end{vmatrix}
= 0
\]
then Show that  \(xyz=1\)

---

**UNIT TEST 3**

**PERMUTATION TEST**

**PART A**

Choose the correct answer

Answer all the questions

1 x 30 = 30
1. If $x! = 24$ the value of $x$ is ____________
   (i) 4 (ii) 3 (iii) 4! (iv) 1

2. The value of $3! + 2! + 1! + 0!$ is ____________
   (i) 10 (ii) 6 (iii) 7 (iv) 9

3. The value of $100C_r = 100C_{4r}$ is ____________
   (i) 25 (ii) 24 (iii) 4 (iv) 5

4. The total number of ways of analyzing 6 persons around a table is ________
   (i) 6 (ii) 5 (iii) 6! (iv) 5!

5. The value of $x(x-1)(x-2)!$ is ____________
   (i) $x!$ (ii) $(x-1)!$ (iii) $(x-2)!$ (iv) $(x+1)!$

6. 2 persons can occupy 7 places in ________ ways
   (i) 42 (ii) 14 (iii) 21 (iv) 7

7. The value of $8P_3$ is ____________
   (i) $8 \times 7 \times 6$ (ii) $\frac{8 \times 7 \times 6}{3 \times 2 \times 1}$ (iii) $8 \times 7$ (iv) $3 \times 2 \times 1$

8. The value of $^8C_0$ is ____________
   (i) 8 (ii) 1 (iii) 7 (iv) 0

9. The value of $^{10}C_9$ is ____________
   (i) 9 (ii) 1 (iii) $^{10}C_1$ (iv) 0

10. Number of lines that can be drawn using 3 points in which none of 5 points are collinear is ____________
    (i) 10 (ii) 20 (iii) 5 (iv) 1

11. If $^{10}C_r = ^{10}C_{4r}$ then the value of $r$ is ____________
    (i) 2 (ii) 4 (iii) 10 (iv) 1

12. If $^5C_x + ^5C_4 = ^5C_5$ then the value of $x$ is ____________
    (i) 2 (ii) 4 (iii) 10 (iv) 1

13. If $^nC_4 + ^nC_5 = ^nC_5$ the value of $n$ is ____________
    (i) 10 (ii) 5 (iii) 9 (iv) 8

14. If $^nP_r = 720^nC_r$ then the value of $r$ is ____________
    (i) 6 (ii) 5 (iii) 4 (iv) 7

15. The number of arrangements that can be made out of letters of word ENGINEERING
    (i) $11!$ (ii) $\frac{11!}{(3!)(2!)(2!)}$ (iii) $\frac{11!}{(3!)(2!)}$ (iv) $\frac{11!}{3!}$

16. The number of 4 digit numbers, that can be formed by the digits 3,4,5,6,7,8,0 and no digit is being repeated, is ________
    (i) 720 (ii) 840 (iii) 280 (iv) 560

17. A Polygon has 44 diagonal then the number of its sides is ____________
    (i) 28 (ii) 48 (iii) 20 (iv) 42

18. 20 persons are invited for a party. The number of ways in which they and the host can be seated at a circular table is two particular persons be seated on either side of the host is equal to
    (i) 18! 2! (ii) 18! 3! (iii) 19! 2! (iv) 20! 2!

19. The number of permutation of the letters of the word APPLE are ____________
    (i) 120 (ii) 60 (iii) 30 (iv) 5

20. In how many ways can 2 different balls be distributed among 3 boxes
    (i) 9 (ii) 2 (iii) 3 (iv) 6

21. The number of permutation of $n$ distinct objects is ____________
    (i) $n-1$ (ii) $n$ (iii) $(n-1)!$ (iv) $n!$

22. The number of linear permutation of $n$ object is ____________
    (i) $n-1$ (ii) $n$ (iii) $(n-1)!$ (iv) $n!$
23. There are n things and if the direction is not taken into consideration, the number of circular
permutation is ____________
(i) (n-1)!  (ii) n  (iii) \(\frac{(n-1)!}{2}\)  (iv) n!

24. In how many ways can a garland of 20 flowers are made ?
(i) 19!  (ii) 20!  (iii) \(\frac{19!}{2}\)  (iv) 20

25. From a group of 15 cricket players, a team of 11 players is to be chosen. In how many ways this
can be done?
(i) 1365  (ii) 3251  (iii) 5261  (iv) 1653

26. The number of triangles that can be formed by joining the vertices of a hexagon?
(i) 20  (ii) 10  (iii) 15  (iv) 35

27. The number of diagonals of a hexagon
(i) 20  (ii) 10  (iii) 15  (iv) 35

28. If \(^nC_8 = ^nC_6\) then the value of \(^nC_2\) is ____________
(i) 14  (ii) 8  (iii) 6  (iv) 91

29. If \(^{100}C_r = ^{100}C_{4r}\) the value of r is ____________
(i) 25  (ii) 20  (iii) 50  (iv) 100

30. In how many ways can 7 identical beads be strung into a ring
(i) 630  (ii) 7!  (iii) 360  (iv) 6!

31. How many diagonals are there in a polygon?
32. In how many ways can five children stand in a queue?
33. In how many ways can a garland of 20 flowers are made?
34. In how many ways can 8 students be seated in a circle?
35. How many different words can be formed with letters of the word “HARYANA”?
36. A family of 4 brothers and 3 sisters is to be arranged in a row, for a photograph. In how many
ways can they be seated, if
(i) all the sisters sit together
(ii) all the sisters are not together.
37. If \(^9P_r = 3024\) find r
38. If \(^{n-1}P_3 : ^nP_4 = 1:9\) find n
39. How many numbers divisible by 5 and lying between 5000 and 6000 can be formed from the
digits 5,6,7,8 and 9?
40. How many odd numbers less than 1000 can be formed by using the digits 0,3,5,7 when repetition
of digits is not allowed?
41. In how many ways can five children stand in a queue?

Answer any 10 questions 10 x 3 = 30

Answer any 4 questions 4 x 10 = 40
UNIT TEST 4
BINOMIAL THEORM AND MATHEMATICAL INDUCTION

Part A

Answer all the questions all

5 x1 =5

1. The total number of terms in the expansion of \((x+a)^4\)
   a) 4 b) 3 c) 2 d) 5

2. The second term in the expansion of \((2x+1)^2\)
   a) 12x\(^2\) b) 8x\(^3\) c) 6x d) 1

3. The last term in the binomial coefficient \((x+a)^n\)
   a) \(a^n\) b) \(x^n\) c) \(x\) d) \(a\)

4. The middle term in the expansion of \((a-b)^8\)
   a) 4 b) 3 c) 6 d) 5

5. The sum of the binomial coefficient of \((x+a)^n\) is
   a) \(2^n\) b) \(2^{n-1}\) c) \(2^{n+1}\) d) None

Part B

7 x 5 = 35

6. Expand \((2x+3y)^5\)

7. Evaluate using Binomial theorem \(101^3\)

8. Simplify \((\sqrt{2} + 1)^5 + (\sqrt{2} - 1)^5\)

9. Find the coefficient of \(x^5\) in the expansion of \(\left(\frac{x}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)^6\)

10. Find the term independent of \(\frac{x}{\sqrt{2}}\)

11. Evaluate the 7th power of 11 using Binomial theorem

12. Write the expansion of \((a-2b)^5\)

13. Prove the following by the principle of mathematical induction
   \(5^n - 1\) is divisible by 24 for all \(n \in N\)

14. Prove the following by the principle of mathematical induction
   \(4+8+12+\ldots+4n = 2n(n+1)\)

Part C

6 x 10 = 60

15. The first 3 terms in the expansion of \((1-ax)^n\) are 1-14x+84x\(^2\) find \(a\) and \(n\)

16. Prove by induction method that \(1+4+7+\ldots+\frac{n(3n-1)}{2}\)

17. If the 21st term and 22nd term in the expansion of \((1+x)^{44}\) are equal find the value of \(x\)

18. In the expansion of \((1+x)^{20}\), the coefficient of \(r\)th and \((r+1)\)th terms are in the ratio 1:6 find the value of \(r\)

19. Prove the following by the principle of mathematical induction \(a^n - b^n\) is divisible by \(a-b\)

20. Prove the following by the principle of mathematical induction The sum \(S_n = n^3+3n^2+5n+3\) is divisible by 3 for all \(n \in N\)

21. Prove by induction method that \(7^{2n} + 16n - 1\) is divisible by 64
22. Find the middle term in the expansion of \( \frac{2x^2y^3}{3} \).

UNIT TEST 5
SEQUENCE AND SERIES
Part B
Answer any 10 questions 10 x 4 = 40

1. Find the 7th term of the H.P \( \frac{1}{5}, \frac{1}{9}, \frac{1}{13} \) .......

2. Find the 4th term and 7th term of the H.P \( \frac{1}{2}, \frac{4}{13}, \frac{2}{9} \) .......

3. The 9th term of an H. P is \( \frac{1}{465} \) and the 20th term is \( \frac{1}{388} \). Find the 40th term.

4. If a, b, c are in G.P. Prove that \( \log_m a, \log_m b, \log_m c \) are in H.P.

5. Insert 5 arithmetic mean between 1 and 19.

6. The arithmetic mean of two numbers is 34 and their geometric mean is 16. Find the two numbers.

7. The first and second terms of a H.P. are \( \frac{1}{3} \) and \( \frac{1}{5} \) respectively. Find the 9th term.

8. If a, b, c are in H.P. Find that \( \frac{b + a}{b - a} + \frac{b + c}{b - c} = 2 \).

9. The difference between two positive numbers is 18 and 4 times their G.M is equal to 5 times their H.M. Find the numbers.

10. If the A.M. between two numbers is 1, prove that their H.M. is the square of their G.M.

11. If a, b, c are in A.P.; and a, mb, c are in G.P then prove that, \( a, m^2b, c \) are in H.P.

12. If the pth and qth terms of a H.P are \( q \) and \( p \) respectively, show that \( (pq) \)th term is 1.

Part C
Answer any 6 questions 6 x 10 = 60

13. Write the first four terms in the expansion of \( \left( \frac{1}{\sqrt{6 - 3x}} \right) \) where \( |x| < 2 \).

14. Find the 5th term in the expansion of \( \left( 1 - 2x^3 \right)^{11/2} \).

15. Evaluate \( \frac{1}{\sqrt{128}} \) correct to four places of decimal.

16. Find the value of \( \sqrt{126} \) correct to four decimal places.

17. If \( x \) is so small prove that \( \frac{(4x + 3)\sqrt{9 + x}}{6 + 9x} = \frac{3 - x}{6} \).

18. If \( x \) is so small show that \( \sqrt{1 - x} = 1 - x + \frac{x^2}{2} \) (approx.)

19. If \( x \) is large and positive show that \( \sqrt{x^3 + 6} - \sqrt{x^3 + 3} = \frac{1}{x^2} \) (approx.)

20. If \( x \) is so large prove that \( \sqrt{x^2 + 25} - \sqrt{x^2 + 9} \) nearly 8.
TRIGONOMETRY
Part A

1. An angle between 0° and -90° has its terminal side in

\[ \frac{1}{360} \]

of a complete rotation clockwise is

(i) -1°  (ii) -360°  (iii) -90°  (iv) 1°

2. If the terminal side is collinear with the initial side in the opposite direction then the angle included is

(i) 0°  (ii) 90°  (iii) 180°  (iv) 270°

3. If \( p \cosec \theta = \cot 45° \) then \( p \) is __________

(i) \( \cos 45° \)  (ii) \( \tan 45° \)  (iii) \( \sin 45° \)  (iv) \( \sin \theta \)

4. \( \frac{1}{360} - \frac{1}{\sin^2 \theta} = \frac{\cot \theta}{\cosec \theta} \)

(i) 0  (ii) 1  (iii) \( \cot \theta - \sin^2 \theta \)  (iv) \( \sin^2 \theta - \cot^2 \theta \)

5. \( (\sin 60° + \cos 60°)^2 + (\sin 60° - \cos 60°)^2 = \) ______________

(i) 3  (ii) 1  (iii) 2  (d) 0

6. \( \frac{1}{\sec 60° - \tan 60°} = \)

(i) \( \frac{\sqrt{3} + 2}{2\sqrt{3}} \)  (ii) \( \frac{\sqrt{3} - 2}{2\sqrt{3}} \)  (iii) \( \frac{1 + \sqrt{3}}{2} \)  (iv) \( \frac{1 - \sqrt{3}}{2} \)

7. If \( x = a \cos^3 \theta \); \( y = b \sin^3 \theta \) then \( \left( \frac{x^2}{a^2} \right) + \left( \frac{y^2}{b^2} \right) \) is equal to __________

(i) 2 \( \cos^3 \theta \)  (ii) 3b \( \sin^3 \theta \)  (iii) 1  (iv) \( a \ b \sin^2 \theta \ \cos^2 \theta \)

8. The value of \( \frac{1}{\sec (60°)} \) is

(i) \( \frac{1}{2} \)  (ii) -2  (iii) 2  (iv) -\( \frac{1}{2} \)

9. \( \sin (90° + \theta) \ \sec (360° - \theta) = \) ______________

(i) \( \cosec \theta \)  (ii) -\( \cosec \theta \)  (iii) \( \cosec \theta \)  (iv) -\( \sec \theta \)

10. When \( \sin A = \frac{1}{\sqrt{2}} \), between 0° and 360° the two values of A are

(i) 60° and 135°  (ii) 135° and 45°  (iii) 135° and 175°  (iv) 45° and 225°

11. If \( \cos (2n\pi + \theta) = \sin \alpha \) then __________

(i) \( \theta - \alpha \)  (ii) \( \theta = \alpha \)  (iii) \( \theta - \alpha = 90° \)  (iv) \( \alpha - \theta = 90° \)

12. \( \frac{\tan 15° - \tan 75°}{1 + \tan 15° \tan 75°} \) is equal to __________

(i) \( \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \)  (ii) \( \frac{1 + 2\sqrt{3}}{1 - 2\sqrt{3}} \)  (iii) \( \frac{\sqrt{3} - 1}{1 - \sqrt{3}} \)  (iv) 1

13. The value of \( \tan 435° \) is __________

(i) \( \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \)  (ii) \( \frac{1 + \sqrt{3}}{\sqrt{3} - 1} \)  (iii) \( \frac{1 - \sqrt{3}}{1 - \sqrt{3}} \)  (iv) 1
14. The value of $\cos 9^\circ \cos 6^\circ - \sin 9^\circ \sin 6^\circ$ is equal to __________
   (i) 0  (ii) $\frac{\sqrt{3}+1}{4}$  (iii) $\sin 75^\circ$  (iv) $\sin 15^\circ$

15. $\tan \left( \frac{\pi}{4} + x \right)$ is __________
   (i) $\frac{1 + \tan x}{1 - \tan x}$  (ii) $\tan x + \tan x$  (iii) $-\tan x$  (iv) $\tan \frac{\pi}{4}

16. In a triangle $\triangle ABC$ if $\cot (A+B) = 1$ then $\tan C$ is __________
   (i) 0  (ii) 1  (iii) $\infty$  (iv) $-1$

17. If $\sin A = 1$ then $\sin 2A$ is equal to __________
   (i) 2  (ii) 1  (iii) 0  (iv) $-1$

18. The value of $\sin 54^\circ$ is __________
   (i) $\frac{1 - \sqrt{5}}{4}$  (ii) $\frac{\sqrt{5} - 1}{4}$  (iii) $\frac{\sqrt{5} + 1}{4}$  (iv) $-\frac{\sqrt{5} - 1}{4}$

19. The value of $\frac{1 - \cos 15^\circ}{1 + \cos 15^\circ} = __________$
   (i) $\sec 30^\circ$  (ii) $\tan \frac{\sqrt{15}}{2}$  (iii) $\tan 30^\circ$  (iv) $\tan^2 \frac{1}{2}

Answer any 5 questions

Part B

5 x 6 = 30

20. Without using the tables, prove that $\sin 780^\circ \sin 480^\circ + \cos 120^\circ \cos 60^\circ = \frac{1}{2}$

21. If $\cos \theta = -\frac{1}{2}$ and $\tan \theta > 0$ show that $\frac{5 \tan \theta + 4 \sin \theta}{\sqrt{3} \cos \theta - 3 \sin \theta} = 3$

22. If $A, B, C, D$ are angles of a cyclic quadrilateral prove that $\cos A + \cos B + \cos C + \cos D = 0$

23. If $a \sin^2 \theta + b \cos^2 \theta = c$, show that $\tan^2 \theta = \frac{c - b}{a - c}$

24. If $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$ show that $x^2 + y^2 = a^2 + b^2$

25. If $\tan A = \frac{1 - \cos B}{\sin B}$ prove that $\tan 2A = \tan B$ where $A$ and $B$ are acute angles

26. Prove that $(1 + \cot A + \tan A)(\sin A - \cos A) = \sec A \cos^2 A - \csc A$

27. Prove that $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$
28. Given \( p = \tan \theta + \sin \theta, \tan \theta - \sin \theta \) and \( q > p \) then Show that \( p^2 - q^2 = 4 \sqrt{pq} \)

29. If \( \cos \theta + \sin \theta = \sqrt{2} \cos \theta, \) Show that \( \cos \theta - \sin \theta = \sqrt{2} \sin \theta \)

30. If \( A, B \) are acute angles, \( \sin A = \frac{3}{5}, \cos B = \frac{12}{13} \) find \( \cos (A+B) \)

31. If \( \sin A = \frac{1}{3}, \sin B = \frac{1}{4} \) find \( \sin (A+B) \)

32. If \( \tan A = 3 \) and \( \tan B = \frac{1}{2} \) Prove that \( A - B = \frac{\pi}{4} \)

33. If \( \cos (\alpha - \beta) = \frac{4}{5} \) and \( \sin (\alpha - \beta) = \frac{5}{13} \) Find \( \tan 2\alpha \)

34. If \( A + B = 45^\circ \) Show that \( (1 + \tan A)(1 + \tan B) = 2 \) and hence deduce the value of \( \tan 122^\circ \)

35. If \( A + B + C = \pi \)
   (i) Prove that \( \tan A + \tan B + \tan C = \tan A \tan B \tan C \)
   (ii) Prove that \( \tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C \)

36. Find the value of \( \sin 18^\circ \) and hence deduce the value of \( \cos 36^\circ \)

37. If \( \tan \alpha = \frac{1}{3} \) and \( \tan \beta = \frac{1}{7} \) Show that \( 2\alpha + \beta = \frac{\pi}{4} \)

38. Prove that \( \cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8} \)

UNIT TEST 7
ANALYTICAL GEOMETRY

1. The angle made by the line \( x+y+7=0 \) with the positive direction of \( x \) axis is __________
   
   a) \( 45^\circ \)  b) \( 135^\circ \)  c) \( 210^\circ \)  d) \( 60^\circ \)

3) The slope of the line \( 3x - 5y + 8 = 0 \) is __________
   
   a) \( \frac{3}{5} \)  b) \( -\frac{3}{5} \)  c) \( \frac{5}{3} \)  d) \( -\frac{5}{3} \)

4) Two lines \( ax + by + c = 0 \) and \( px + qy + r = 0 \) are perpendicular if __________
   
   a) \( \frac{a}{p} = \frac{b}{q} \)  b) \( \frac{a}{b} = \frac{q}{p} \)  c) \( \frac{a}{b} = -\frac{p}{q} \)  d) \( \frac{a}{b} = -\frac{q}{p} \)

5) Slope of the line perpendicular to \( ax + by + c = 0 \) is __________
   
   a) \( \frac{-a}{b} \)  b) \( \frac{-b}{a} \)  c) \( \frac{b}{a} \)  d) \( \frac{a}{b} \)

8) When \( ax + 3y + 5 = 0 \) and \( 2x + 6y + 7 = 0 \) are parallel then the value of \( "a" \) is __________
   
   a) \( 2 \)  b) \( -2 \)  c) \( 1 \)  d) \( 6 \)

9) The value of \( "a" \) for which \( 2x + 3y - 7 = 0 \) and \( 3x + ay + 5 = 0 \) are parallel is __________
Perpendicular is ______________

10) The Value of “m” for which the point (2,3) lies on the line 2x-my+11 = 0 is
   a) -5  b) 0  c) 3  d) -3

11) The slope of a line is -1, then the angle of inclination is ______________
   a) 135°  b) 120°  c) 180°  d) 150°

12) The point of intersection of x + y = 3 and x - y =1 is ______________
   a) (1,1)  b) (1,0)  c) (2,1)  d) (1,-1)

13) When does the line 4x + 3y – 12 = 0 meet the x axis?
   a) (3,0)  b) (0,3)  c) (0,4)  d) (4,0)

14) The line 2x + 3y + 1= 0 and m x + 2y + 7 = 0 are perpendicular then
   The value of m is ______________
   a) 1  b) -1  c) 2  d) -3

15) The slope of the line 2x – 3y + 1 = 0 is
   a) 2/3  b) 3/2  c) -3/2  d) -2/3

16) Which of the following has the greatest y intercept in magnitude ?
   a) 2x+3y=4  b) x+2y=3  c) 3x + 4y = 5  d) 4x+5y=6

17) If the equation of the straight line is $y = \sqrt{3}x + 4$ , then the angle made by the
    Straight line with the positive direction of x axis is
   a) 45°  b) 30°  c) 60°  d) 90°

18) If the straight line $a_1x+b_1y+c=0$ and $a_2x+b_2y+c_2 = 0$ are perpendicular , then
   a) $\frac{a_1}{a_2} = -\frac{b_1}{b_2}$  b) $\frac{a_1}{b_1} = \frac{a_2}{b_2}$  c) $a_1a_2 = -b_1b_2$  d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

19) Which of the following is a parallel line to 3x+4y+5=0 ?
   a) 4x+3y+6=0  b) 3x-4y+6=0  c) 4x-3y+9=0  d) 3x +4y+6=0

20) Which of the following is the equation of a straight line that is neither
    parallel nor perpendicular to the straight line given by x +y=0 ?
   a) y=x  b) y-x+2=0  c) 2y=4x+1  d) y+x+2=0

Part B

21. A and B are two points (1,0) and (-2,3) . Find the equation of the locus of a
   point such that PA = 4 PB

22. If the point P ( 5t-4, t+1 ) lies on the line 7x-4y+1= 0 Find the value of t
    Find also the co – ordinates of P

23. Find the equation of the straight line passing through the point (1,2) and making intercepts on the
    co ordinate axes which are in the ratio 2: 3

24. Find the length of the perpendicular from (2,-3) to the line  2x-y+9=0

25. Find the angle between the straight lines  3x – 2y +9 =0 and 2x+y-9=0

26. Find the equation of the straight line passing through the point (1,-2) and parallel to  The
    straight line 3x+2y-7=0

27. Find the distance between the parallel lines  2x+y-9=0 and 4x+2y+7=0
28. Find the equation of the straight line joining the point (4, -3) and the intersection of the lines
   \[2x - y + 7 = 0 \text{ and } x + y - 1 = 0\]

29. For what value of \( m \) the three straight lines \(3x + y + 2 = 0, 2x - y + 3 = 0\) and \(x + my - 3 = 0\) are concurrent?

30. Find the equation of the straight line passing through the intersection of the straight lines
   \[2x + y = 8 \text{ and } 3x - y = 2\] and through the point (2, -3)

31. Find the co-ordinates of the orthocenter of the triangle whose vertices are the points (-2, -1), (6, -1), and (2, 5)

32. Find the equation of the straight line passing through the intersection of the lines \(5x - 6y = 1\) and \(3x + 2y + 5 = 0\) and perpendicular to the line \(3x - 5y + 11 = 0\)

33. Find the values of \( p \) for which the straight lines \(8px + (2 - 3p)y + 1 = 0\) and \(px + 8y - 7 = 0\) are perpendicular to each other

34. Find the point on the y axis whose perpendicular distance from the straight line \(4x - 3y - 12 = 0\)

35. Find the co-ordinates of the points on the straight line \(y = x + 1\) which are at a Distance of 5 units from the straight line \(4x - 3y + 20 = 0\)

36. A and B are the two points (1, 0) and (-2, 3). Find the equation of the locus of a Point such that \(PA^2 + PB^2 = 10\)

UNIT TEST 8
PAIR OF STRAIGHT LINES
Part B

Answer any 8 questions 8 x 5 = 40

1. Find the angle between the straight lines \(x^2 + 4xy + 3y^2 = 0\)

2. Find the angle between the pair of straight line \((a^2 - 3b^2)x^2 + 8abxy + (b^2 - 3a^2)y^2 = 0\)

3. Show that if one of the angles between pair of straight lines then \((a + 3b)(3a + b) = 4h^2\)

4. The slope of one of the straight lines of \(ax^2 + 2hxy + by^2 = 0\) thrice that of the other, Show that \(3h^2 = 4ab\)

5. The slope of one of the straight lines \(ax^2 + 2hxy + by^2 = 0\) is twice that of the other, Show that \(8h^2 = 9ab\)

6. Show that \(x^2 - y^2 + x - 3y - 2 = 0\) represents a pair of straight lines. Also find the angle between them

7. Show that the equation \(3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0\) represents a pair of straight lines and also find the angle between the straight lines

8. Show that \(3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0\) represents a pair of straight lines and the angle between them is \(\tan^{-1}\left(\frac{2}{11}\right)\)
9. Combined the equation of the straight lines whose separate equations are \( x + 2y - 3 = 0 \) and \( 3x - y + 4 = 0 \)

10. Find the combined equation of the straight lines whose separate equations are \( x + 2y - 3 = 0 \) and \( 3x + y + 5 = 0 \)

**Part C**

Answer any 5 questions \(5 \times 10 = 50\)

11. For what value of \( k \) does \( 12x^2 + 7xy + ky^2 + 13x - y + 3 = 0 \) represents a pair of straight lines? Also write the separate equations.

12. If the equation \( 12x^2 - 10xy + 2y^2 + 14x - 5y + c = 0 \) represents a pair of straight lines, find the value of \( c \). Find the separate equations of the straight lines and also the angle between them.

13. If the equation \( ax^2 + 3xy - 2y^2 - 5x + 5y + c = 0 \) represents a pair of perpendicular straight lines, find the value of \( a \) and \( c \).

14. Show that \( 9x^2 + 24xy + 16y^2 + 21x + 28y + 6 = 0 \) represents a pair of parallel straight lines and find the distance between them.

15. Show that the equation \( 4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0 \) represents a pair of parallel lines and find the distance between them.

16. Find the combined equation of the straight lines through the origin, one of which is parallel to and the other is perpendicular to the straight line \( 2x + y + 1 = 0 \).

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**UNIT TEST 9**

**CIRCLES**

**Part A**

Answer all the questions \(1 \times 20 = 20\)

1. The center of the circle \( x^2 + y^2 + 6y - 9 = 0 \) is ____________
   a) \((0,3)\)  
   b) \((0,-3)\)  
   c) \((3,0)\)  
   d) \((-3,0)\)

2. The equation of the circle with center at \((0,0)\) and radius 3 units is ____________
   a) \(x^2 + y^2 = 3\)  
   b) \(x^2 + y^2 = 9\)  
   c) \(x^2 + y^2 = \sqrt{3}\)  
   d) \(x^2 + y^2 = 3\sqrt{3}\)

3. The length of the diameter of a circle with center \((1,2)\) and passing through the Point \((5,5)\) is ____________
   a) 5  
   b) \(\sqrt{45}\)  
   c) 10  
   d) \(\sqrt{56}\)

4. If \((1,-3)\) is the center of the circle \(x^2 + y^2 + ax + by + 9 = 0\) its radius is ____________
   a) \(\sqrt{10}\)  
   b) 1  
   c) 5  
   d) \(\sqrt{19}\)

5. The area of the circle \((x-2)^2 + (y-4)^2 = 25\) is ____________
   a) 25  
   b) 5  
   c) 10  
   d) \(25\pi\)

6. The equation of a tangent at \((1,2)\) to the circle \(x^2 + y^2 = 5\) is ____________
   a) \(x + y = 5\)  
   b) \(x + 2y = 5\)  
   c) \(x - y = 5\)  
   d) \(x - 2y = 5\)

7. The length of tangent from \((3,4)\) to the circle \(x^2 + y^2 - 4x + 6y - 1 = 0\) is ____________
   a) 7  
   b) 6  
   c) 5  
   d) 8

8. If \(y = 2x + c\) is a tangent to the circle \(x^2 + y^2 = 5\) then the value of \(c\) is ____________
   a) \(\pm \sqrt{5}\)  
   b) \(\pm 25\)  
   c) \(\pm 5\)  
   d) \(\pm 2\)

9. The center of the circle \(2x^2 + 2y^2 + 6x + 8y - 41 = 0\) is ____________
   a) \((-6,-8)\)  
   b) \((6,8)\)  
   c) \((-3,-4)\)  
   d) \((3,4)\)
10. The length of the tangent from (1,2) to the circle \( x^2+y^2+x+2y+5=0 \) is ________
   a) 15  b) \( \sqrt{15} \)  c) \( \sqrt{10} \)  d)10

11. The radius of the circle \( x^2+y^2+6x-8y=0 \) is ________
   a) 25  b) 0  c) 5  d) \( \sqrt{5} \)

12. The length of the diameter of the circle with center (2,1) and passing through the Point (-2,1)is ________
   a) 4  b) 8  c) 4 \( \sqrt{5} \)  d) 2

13. Given that (1,-1) is the center of the circle \( x^2+y^2+ax+by-9=0 \) Its radius is ________
   a) 3  b) \( \sqrt{2} \)  c) \( \sqrt{11} \)  d) 11

14. The equation of a circle with centre (0,0) and passing through the point (5,0) is ________
   a) \( x^2+y^2-10x=0 \)  b) \( x^2+y^2=25 \)  c) \( x^2+y^2+10x=0 \)  d) \( x^2+y^2-10y=0 \)

15. The radius of the circle \( x^2+y^2-2x+4y-5=0 \) is ________
   a) 1  b) \( \sqrt{2} \)  c) \( \sqrt{3} \)  d) \( \sqrt{4} \)

16. The center of the circle \( x^2+y^2+2x-4y-4=0 \) is ________
   a) (2,4)  b) (1,2)  c) (-1,2)  d) (-2,-4)

17. If \( 2x+3y=0 \) and \( 3x-2y=0 \) are the equations of two diameters of a circle, then its center is ________
   a) (1,-2)  b) (2,3)  c) (0,0)  d) (-3,2)

18. If the line \( y=2x-c \) is a tangent to the circle \( x^2+y^2=5 \), then the value of \( c \) is ________
   a) \( \pm 5 \)  b) \( \pm \sqrt{5} \)  c) \( \pm \sqrt{5} \)  d) \( r_1+r_2 \)

19. The length of the tangent from (4,5) to the circle \( x^2+y^2=25 \) is ________
   a) 5  b) 4  c) 25  d) 16

20. Which of the following point lies inside the circle \( x^2+y^2+2x-4y-5=0 \) ________
   a) (5,10)  b) (-5,7)  c) (9,0)  d) (1,1)

Part B

Answer any 5 question 5 x 6 = 30

21. For what values of \( a \) and \( b \) does the equation \( (a-2)x^2+by^2+(b-2)xy+4x+4y=0 \) represents a circle? Write down the resulting equation of the circle.

22. \( x+2y=7, \ 2x+y=8 \) are two diameters of a circle with radius 5 units find the equation of the circle

23. Find the Cartesian equation of the circle whose parametric equation are 
   \( x = 2\cos{\theta} \) and \( y = 2\sin{\theta} \) \( 0 \leq \theta \leq 2\pi \)

24. Find the equation of tangent to the circle \( x^2+y^2-4x+8y-5=0 \) at (2,1)

25. Find the value of \( p \) if the line \( 3x+4y-p=0 \) is a tangent to the circle \( x^2+y^2=64 \)

26. Show that the circles \( x^2+y^2-2x+6y+6=0 \) and \( x^2+y^2-5x-6y+15=0 \) touch each other

27. Prove that the circles \( x^2+y^2-8x+6y-23=0 \) and \( x^2+y^2-2x-5y+16=0 \) are orthogonal

Part C

Answer any 5 question 5 x 10 = 50

28. Find the circles which cuts orthogonally each of the circles 
   \( x^2+y^2+2x+4y+1=0, \ x^2+y^2-4x+3=0 \) and \( x^2+y^2+6y+5=0 \)

29. Find the length of the chord intercepted by the circle
\[ x^2 + y^2 - 2x - y + 1 = 0 \] and the line \[ x - 2y = 1 \]

30. Find the equation of the circle passing through the points (1,1) (2,-1) and (3,2)

31. Find the equation of the circle whose centre is on the line \( x = 2y \) and which passes through the points (-1,2) and (3,-2)

32. Find the co-ordinates of the point of intersection of the line \( x + y = 2 \) with the circle \( x^2 + y^2 = 4 \)

33. Find the equation of the circle, which is concentric with the circle \( x^2 + y^2 - 4x - 6y - 9 = 0 \) and passing through the point (-4,-5)

**Unit test 10**

**FUNCTIONS AND GRAPHS**

**Part A**

Answer all the questions

1. __________ was the first scientist who gave the functions concept
   1) Leonard Euler  2) Democritus  3) Leibniz  4) Archimedes

2. \((-\infty, \infty)\) is called
   1) Closed interval  2) finite interval  3) open interval  4) semi open interval

3. In the space the neighborhood of a point defined as
   1) sphere  2) open disc  3) open interval  4) closed interval

4. When \( A = \pi r^2 \), \( A \) is called __________ variable
   1) independent  2) dependent  3) independent as well as dependent  4) independent or dependent

5. \( f(x) = \sqrt{x} \) the name of the functions is called __________ function
   1) Trigonometry  2) square root  3) exponential  4) logarithm

6. Given \( f(x) = 2x + 1 \) \( x \in \mathbb{R} \) then \( f^{-1}(x) \) is
   1) \( \frac{x+1}{2} \)  2) \( \frac{x-1}{2} \)  3) \( \frac{1}{2x+1} \)  4) \( \frac{1}{2x-1} \)

7. Composition of any two function is always
   1) Non commutative  2) commutative  3) not commutative  4) None

8. \( f(x) \) is said to be invertible function only if
   1) \( f \circ f^{-1} = I \)  2) \( fo f^{-1} = 0 \)  3) \( fo f^{-1} = 1 \)  4) \( fo f^{-1} = 0 \)

9. The graph of the identity function is a
   1) st. line passing through origin  2) circle  3) parabola  4) ellipse

10. The graph of the Linear function is a
    1) Straight line  2) point  3) Circle  4) parabola

11. The inverse of exponential function is a __________ function
    1) Identity  2) Linear  3) logarithmic  4) exponential

12. The domain \((0, \infty)\) of a logarithmic function becomes the co domain of the
    1) Identity  2) Linear  3) logarithmic  4) exponential

13. \( g: \mathbb{R}^+ \to \mathbb{R} \) defined by \( g(x) = \frac{1}{2} \) then \( g(x) \) is called __________ function
    1) reciprocal  2) inverse  3) invertible  4) identity

14. The graph of the reciprocal function \( f(x) = \frac{1}{x} \) is a
    1) Hyperbola  2) Straight line  3) Rectangular hyperbola  4) Circle

15. The range of Signum function is
    1) \( \mathbb{R} \)  2) \( \mathbb{R} \)  3) \( \mathbb{R} \)  4) \( \mathbb{R} \)
1. The number of type of trigonometrically function are
   1) 2  2) 3  3) 4  4) 1

17. The principle domain of $f(x) = \sin x$ is
   1) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$  2) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$  3) $[0, \pi]$  4) $[-\pi, \pi]$

18. The inequality $x^2 - 7x + 6 > 0$ the value of $x$ lies
   1) out side of (1,6)  2) inside of (1,6)  3) out side of [1,6]  4) out side of [1,6]

19. The solution set of $4x^2 - 25 \geq 0$ is
   1) $(-\infty, -\frac{5}{2}] \cup [\frac{5}{2}, \infty)$  2) $(-\infty, -\frac{5}{2}] \cap [\frac{5}{2}, \infty)$  3) $(-\infty, -\frac{5}{2}] \cap (\frac{5}{2}, \infty)$  4) $(-\infty, -\frac{5}{2}] \cup (\frac{5}{2}, \infty])$

20. The inequality $64x^2 + 48x + 9 < 0$
   1) $x = -\frac{3}{8}$  2) $x = \frac{3}{8}$  3) $x = -\frac{8}{3}$  4) no solution

**Part B**

Answer any 5 the questions  5 x 6 =30

21. Show that the graph of $x^2 + y^2 = 4$ is not the graph of the function

22. Show that the function $y = x^2$ is not one to one function

23. The two functions $f(x) = x^2 + 1$ and $g(x) = x - 1$ Find fog and gof and Show that they are not equal

24. Let $f$ and $g$ are the two function defined by $f(x) = 2x + 1$ and $g(x) = \frac{x - 1}{2}$
   Show that the two functions are commutative.

25. $f(x)$ is defined by $f(x) = 2x + 1$ find $f^{-1}(x)$

26. Draw the rough diagram of (i) $y = e^x$ and (ii) $\log_e x$

**UNIT TEST 11**

**LIMITS AND CONTINUITY AND DIFFERENTIATION**

**Part A**

Answer all the questions  1 x 10 =10

1. The left limit of $f(x) = \begin{cases} \frac{|x-4|}{x-4}, & \text{for } x \neq 4 \\ 0, & \text{for } x = 4 \end{cases}$
   1) 0  2) 1  3) -1  4) None

2. The value of $\lim_{{x \to 1}} \frac{x^3 - 1}{x - 1}$ is
   1) 3  2) 1  3) 0  4) $\infty$

3. The positive integer $n$ so that $\lim_{{x \to 2}} \frac{x^n - 2^n}{x - 2} = 32$ is
   1) 4  2) 5  3) 2  4) 2

4. The value of $\lim_{{x \to 0}} \frac{1 - \cos x}{x^2}$ is
   1) 1  2) $\frac{1}{2}$  3) 2  4) 0

5. \( \frac{1}{e} = ? \)
   1) 2
   2) 1
   3) 0
   4) None

6. \( \lim_{x \to 0} \frac{a^x - 1}{x} = \) ________
   1) \( \log a \)
   2) 1
   3) \( a^x \)
   4) 0

7. \( \lim_{x \to 0} \frac{5^x - 6^x}{x} = \) ________
   1) \( \log \frac{6}{5} \)
   2) \( \log \frac{5}{6} \)
   3) \( \log \frac{1}{6} \)
   4) \( \log \frac{1}{5} \)

8. \( \lim_{x \to 0} (1 + x)^{\frac{1}{x}} = ? \)
   1) 0
   2) \( e \)
   3) \( \frac{1}{e} \)
   4) 1

9. The Graph of the constant function is
   1) Straight lines parallel to x axis
   2) A straight line perpendicular to x axis
   3) A straight line parallel to y axis
   4) None

10. Which of the following statements are correct
    a. Every differentiable function is continuous
    b. Every continuous function is differentiable
    c. Constant function is not continuous
    d. Exponential function is not continuous

Part B

Answer any 5 questions

6 x 5 = 30

11. Evaluate the left and right limit of \( f(x) = \frac{x^3 - 27}{x - 3} \) at \( x = 3 \) Does the limit of \( f(x) \) as \( x \to 3 \) exists

12. Evaluate \( \lim_{x \to 0} \frac{3^x + 1 - \cos x - e^x}{x} \)

13. Differentiate \( \frac{x^2 - 1}{x^2 + 1} \) with respect to \( x \)

14. Differentiate \( \cos^{-1} \left( \frac{1 - x^2}{1 + x^2} \right) \) with respect to \( x \)

15. Find \( \frac{dy}{dx} \) if \( x = a(\theta + \sin \theta) \) \( y = a(1 - \cos \theta) \)

16. Let \( y = A \cos 4x + B \sin 4x \) Where \( A \) and \( B \) are constants Show that \( y_2 + 16y = 0 \)

Part C

Answer any 6 questions

6 x 10 = 60

17. If \( y = e^{\tan^{-1} x} \) Prove that \( (1 + x^2)y_2 + (2x - 1)y_1 = 0 \)

18. If \( x = \sin t \) \( y = \sin pt \) Show that \( (1-x^2) y_2 - xy_1 + p^2 y = 0 \)

19. Find \( \frac{dy}{dx} \) for \( x^y = y^x \)
20. Differentiate (i) $\tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$ (ii) $\tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$

21. Find the values of $a$ and $b$ so that the functions $f(x) = \begin{cases} 1 & \text{if } x \leq 3 \\ ax + b & \text{if } 3 < x < 5 \\ 7 & \text{if } x \geq 5 \end{cases}$ is continuous at $x=3$ and $x=5$

22. Prove that $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

TEST 12

EACH QUESTION CARRIES 5 MARKS

1. (a) $\int \frac{1}{x^2 + 5x + 7} \, dx$ (b) $\int \frac{dx}{9x + 25}$

2. Evaluate $\int \frac{1}{x^2} \log\left(\frac{1}{x^2}\right) \, dx$

3. $\int \cos^3 x \, dx$

4. Evaluate (i) $\int \frac{1}{1 + 9x^2} \, dx$ (ii) $\int \frac{1}{\sqrt{4x^2 - 25}} \, dx$

5. Evaluate (i) $\int \sec^3 x \tan x \, dx$ (ii) Evaluate $\int x \cos 2x \, dx$

6. Evaluate $\int x e^{-x^2} \, dx$

7. Evaluate $\int \sec^3 x \, dx$

8. Evaluate a) $\int x \log x \, dx$ b) $\int \frac{\cos x}{\sqrt{\sin x}} \, dx$

9. (i) $\int \sin 7x \cos 5x \, dx$ (ii) $\int x \sec^2 x \, dx$

10. Evaluate : $\int \frac{1}{\sqrt{6 - x - x^2}} \, dx$

11. Evaluate : $\int x^2 \log x \, dx$

12. Integrate (a) $\int \sqrt{1 - \sin x} \, dx$ (b) $\int \frac{\log (\tan x)}{\sin 2x} \, dx$

13. Evaluate $\int \frac{x}{5 - 6x - 9x^2} \, dx$

14. Integrate (i) $\int \frac{dx}{e^x + e^{-x}}$ (ii) $\int (2x - 3) \sqrt{x + 1} \, dx$

15. Integrate (i) $\int \cos 3x \sin 4x \, dx$ (ii) $\int \frac{4x + 1}{x^2 + 3x + 1} \, dx$

16. Evaluate $\int \frac{3x + 5}{\sqrt{x^2 - 2x + 3}} \, dx$
17. (i) Prove that \( \int \frac{dx}{\sqrt{x^2 + a^2}} = \log(x + \sqrt{x^2 + a^2}) + c \)  
(ii) \( \int \frac{x^2}{x^3 + 1} \, dx \)

18. Integrate (i) \( \int \sqrt{1 - 3x - x^2} \, dx \)  
(ii) \( \int \frac{dx}{(2x + 3)^2 + 4} \)

19. Evaluate \( \int \frac{4x - 3}{\sqrt{x^2 + 2x - 1}} \, dx \)

20. Prove that \( \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + c \)

UNIT TEST 13  
PROBABILITY ONE MARK TEST  
PART A

Answer any 10 questions  

15 x 1 = 15

1. \( P(A) + \overline{P(A)} = \) is _________
   a) -1  
   b) 0  
   c) 1  
   d) None of these

2. If A and B are mutually exclusive events then \( P(A \cup B) \) is ______
   a) \( P(A) + P(B) \)  
   b) \( P(A) + P(B) - P(A \cap B) \)  
   c) 0  
   d) None

3. The probability of drawing any one spade card from a pack of 52 card is
   a) \( \frac{1}{52} \)  
   b) \( \frac{1}{13} \)  
   c) \( \frac{1}{4} \)  
   d) None

4. The probability of drawing one white ball from a bag containing 6 red, 8 black and 10 yellow balls is ______________
   a) \( \frac{1}{52} \)  
   b) \( 0 \)  
   c) \( \frac{1}{24} \)  
   d) None

5. \( P(\overline{A} \cap B) \) is __________
   a) \( \frac{P(A \cup B)}{P(B)} \)  
   b) \( \frac{P(A \cap B)}{P(B)} \)  
   c) \( \frac{P(A \cap B)}{P(A)} \)  
   d) None

6. If A and B are independent event, then \( P(A \cap B) \) is ______
   a) \( P(A) P(B) \)  
   b) \( P(A) + P(B) \)  
   c) \( \frac{A}{B} \)  
   d) \( P(A) - P(B) \)

7. Probability of sure event is __________
   a) 1  
   b) -1  
   c) 0  
   d) S

8. Probability of an impossible event is _________
   a) 1  
   b) 0  
   c) 2  
   d) \( \phi \)

9. Three coins are tossed once. Find the probability of getting exactly two heads
   a) \( \frac{1}{2} \)  
   b) \( \frac{3}{8} \)  
   c) \( \frac{1}{4} \)  
   d) \( \frac{1}{8} \)

10. A bag contains 5 white and 7 black balls, 2 balls are drawn at random the number of sample space is__________
    1) \( ^{10}C_2 \)  
    2) \( ^{5}C_2 \)  
    3) \( ^{7}C_2 \)  
    4) \( ^{12}C_2 \)

11. A single card is drawn from a pack of 52 cards. the probability that the card is either queen (or) 7
    1) \( \frac{1}{13} \)  
    2) \( \frac{1}{2} \)  
    3) \( \frac{2}{13} \)  
    4) None

12. In box containing 10 bulbs, 2 are defective. What is the probability that among 5 bulbs chosen at random none is defective ?
    1) \( \frac{1}{5} \)  
    2) \( \frac{4}{5} \)  
    3) \( \frac{2}{3} \)  
    4) \( \frac{3}{4} \)

13. The chance that non leap year should have 53 Sundays
    1) \( \frac{1}{7} \)  
    2) \( \frac{2}{7} \)  
    3) \( \frac{3}{7} \)  
    4) \( \frac{4}{7} \)

14. The chance that a leap year should have 53 Mondays
15. If $A \subseteq B$ then
   1) $P(A) < P(B)$     2) $P(A) > P(B)$     3) $P(A) \leq P(B)$     4) $P(A) \geq P(B)$

**Part B**

**Answer any 5 questions**

11. An integer is chosen at random from the first fifty positive integers. What is the probability that the integer is a prime (or) multiple of 4.

12. State and prove addition theorem on Probability

13. The probability that a girl will get an admission in IIT is 0.16 the probability that she will get an admission in Govt. Medical college is 0.24 and the probability that she will get both is 0.11 Find the Probability that she will get only one of the two seats.

14. Two cards are drawn one by one at random from a pack of 52 cards What is the probability of getting two Jacks if (i) the first card is replaced before the second card is drawn (ii) the first card is not replaced before the second card is drawn.

15. A speaks truth in 80% cases and B is 75% cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

16. One bag contains 5 white and 3 black balls Another bag contains 4 white and 6 black balls If one ball is drawn from each bag. Find the probability that (i) both are white (ii) both are black (iii) one white and one black.

**Part C**

**Answer all the questions**

17. The chances of X, Y and Z becoming managers of a certain company are 4:2:3 The probabilities that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5; and 0.4 respectively. What is the probability that Z is appointed as the manager.

18. A problem in Mathematics is given to three students whose chance of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ (i) What is the probability that the problem is solved (ii) what is the probability that exactly one of them will solve it.

19. Given $P(A) = 0.45$ and $P(A \cup B) = 0.75$ Find $P(B)$ if (i) A and B are Mutually Exclusive (ii) A and B are independent $P(A/B) = 0.5$ (iv) $P(B/A) = 0.5$

20. Two cards are drawn from a pack of 52 cards in success find the probability that both are King when (i) the first drawn card is replaced (ii) the card is not replaced

21. A cricket club has 15 members of whom only 5 can bowl. What is the probability that in a team of 11 members at least 3 bowler are selected.

22. State and Prove Multiplication theorem on Probability
TEST 1

Section A

Answer all the questions  1 x 40 =40

1. If \[
\begin{bmatrix}
2 & 1 & 5 \\
3 & 0 & 2 \\
4 & 2 & m \\
\end{bmatrix}
\]
is a singular matrix then the value of m is _________

   a) 1  b) 0  c) 5  d) 10

2. The value of \[
\begin{bmatrix}
2a + b & a \\
2b + c & b \\
2c + a & a \\
\end{bmatrix}
\]
is

   a) 0  b) a+b+c  c) abc  d) 2abc

3. If \[\overrightarrow{OP} = 2\vec{i} + 3\vec{j} - 5\vec{k}\] and the mid point of \[\overrightarrow{PQ} \text{ is } 3\vec{i} - \vec{j} - \vec{k}\] then \[\overrightarrow{OQ}\] is

   a) \[\vec{i} - 4\vec{j} + 4\vec{k}\]  b) \[-\vec{i} + 4\vec{j} - 4\vec{k}\]  c) \[4\vec{i} - 5\vec{j} + 3\vec{k}\]  d) \[\vec{i} + 2\vec{j} - 6\vec{k}\]

4. Given that \[3\vec{i} + 4\vec{j} + \vec{k}\] and \[6\vec{i} + 8\vec{j} + x\vec{k}\] are parallel then the value of x is

   a) 2  b) -2  c) -14  d) 0

5. \[\frac{1}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}\] then A and B are

   a) \(\left(-\frac{1}{2}, \frac{1}{2}\right)\)  b) \(\left(-\frac{1}{2}, -\frac{1}{2}\right)\)  c) \(\left(\frac{1}{2}, \frac{1}{2}\right)\)  d) \(\left(\frac{1}{2}, -\frac{1}{2}\right)\)

6. The value of \[\sum_{i=1}^{n} i^3 \text{ is } \frac{n^2 + 1}{2}\]

   a) 24  b) 336  c) 56  d) 11

7. Number of terms in the expansion of \((2x + 3)^7\) is

   a) infinity  b) 6  c) 7  d) 8

8. Sum of the binomial coefficient is

   a) \(2^a\)  b) \(2^{n+1}\)  c) \(2^n\)  d) \(2^{n-1}\)

9. The 6th term in the given sequence \(a_n = n^2 - 1\) if n is odd, \(\frac{n^2 + 1}{2}\) if n is even

   a) \(\frac{17}{2}\)  b) \(\frac{37}{2}\)  c) \(\frac{5}{2}\)  d) 24

10. The series of \(\log\left(\frac{1+x}{1-x}\right)\) is

    a) \(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\)  b) \(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\)  c) \(2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \ldots\)  d) \(2x + \frac{2x^3}{3!} + \frac{2x^5}{5!} + \ldots\)

11. The sum of 5 Arithmetic mean between two number is _________ times the simple arithmetic mean between them

    a) 25  b) 100  c) 5  d) 10

12. The first term and the third term of HP are \(\frac{1}{3} \text{ and } \frac{1}{7}\) then the section term is

    a) \(\frac{1}{5}\)  b) \(\frac{1}{21}\)  c) \(\frac{1}{10}\)  d) \(\frac{1}{\sqrt{21}}\)

13. The equation of the straight line parallel to \(x - 2y = 4 = 0\) and passing through origin is

    a) \(x - 2y + 1 = 0\)  b) \(x + 2y + 1 = 0\)  c) \(x - 2y = 0\)  d) \(x + 2y = 0\)
14. $y=2x$ and $y=3x$ are two separate lines. The combined equation of their pair of lines is

- a) $y=5x$
- b) $y=6x$
- c) $y^2=6x^2$
- d) $6x^2-5xy+y^2=0$

15. The quadrant in which $-140^\circ$ lies in the ________ quadrant

- a) third
- b) second
- c) first
- d) fourth

16. If (-2,-3) is a point on the terminal side of $\theta$ $\cos \theta$ is _______

- a) $\frac{\sqrt{13}}{3}$
- b) $-\frac{3}{\sqrt{13}}$
- c) $-\frac{2}{\sqrt{13}}$
- d) $\frac{3}{2}$

17. $\sin (-780^\circ)$ is __________

- a) $\frac{\sqrt{3}}{2}$
- b) $-\frac{\sqrt{3}}{2}$
- c) $\frac{1}{2}$
- d) $-\frac{1}{2}$

18. $\tan 15^\circ$ is _______

- a) $2+\sqrt{3}$
- b) $2 - \sqrt{3}$
- c) $\frac{\sqrt{3}}{2} - 2$
- d) $1 - \sqrt{2}$

19. $\frac{\cos 17^\circ + \sin 17^\circ}{\cos 17^\circ - \sin 17^\circ}$ = __________

- a) $\tan 62^\circ$
- b) $\tan 26^\circ$
- c) $\tan 34^\circ$
- d) None

20. $\cos 2A - \cos 4A$ = __________

- a) $\sin 3A \sin A$
- b) $\cos 2A \cos A$
- c) $2 \sin 3A \sin A$
- d) $2 \cos 2A \cos A$

21. The general solution of $\sin \theta = \frac{1}{2}$ is _______

- a) $2n\pi + (-1)^n \frac{\pi}{6}$
- b) $n\pi + \frac{\pi}{6}$
- c) $2n\pi + \frac{\pi}{6}$
- d) $n\pi + (-1)^n \frac{\pi}{6}$

22. If $f(x) = f(-x)$ for all $x$ in the domain then the function is called _______ function

- a) zero
- b) even
- c) odd
- d) identity

23. The range for $y=\cos x$ is _______

- a) $(-\infty, \infty)$
- b) $(0, \pi)$
- c) $(0,1)$
- d) $(-1,1)$

24. The function $f$: $\mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 + 5x^2 + 3$ is a polynomial function of degree _______

- a) 1
- b) 3
- c) 2
- d) 5

25. The domain of the rational function $f(x) = \frac{x^2 + 2x - 3}{x^2 - x}$ is _______

- a) $(0,1)$
- b) $(-3,1)$
- c) $\mathbb{R} - (0,1)$
- d) $\mathbb{R} - (-3,1)$

26. The solution of $x^2 - 5x + 6 > 0$ is

- a) $(2,3)$
- b) $\mathbb{R} - (2,3)$
- c) $(-\infty,2) \cap (3, \infty)$
- d) $(-\infty,2) \cup (3, \infty)$

27. The domain of $y = \cot x$ is _______

- a) $k\pi$
- b) $\mathbb{R} - \{k\pi\}$
- c) $k \frac{\pi}{2}$
- d) $\mathbb{R} - k \frac{\pi}{2}$

28. $\lim_{x \to 0} \frac{e^x - 1}{x}$ is _______

- a) $e^1$
- b) $e$
- c) 0
- d) 1

29. $\lim_{x \to \infty} \frac{x(x+1)(x+2)}{x^2 + 4}$ = _______

- a) 1
- b) $\frac{1}{2}$
- c) infinity
- d) Zero
30. The value of \( a \) if \( \lim_{x \to \infty} \frac{ax^2 - 1}{x^2 + 1} = 1 \) is __________
   a) 0 \hspace{1cm} b) 1 \hspace{1cm} c) -1 \hspace{1cm} d) None

31. The derivative of \( a^x \) is
   a) \( a^x \) \hspace{1cm} b) \( x \cdot a^{x-1} \) \hspace{1cm} c) \( a^x \log a \) \hspace{1cm} d) \( x \cdot \log a \)

32. \( y = \sin (x^2) \) then \( \frac{dy}{dx} \) is
   a) \( 2x \sin (x^2) \) \hspace{1cm} b) \( 2x \cos 2x \) \hspace{1cm} c) \( \cos (x^2) \) \hspace{1cm} d) \( 2x \cos (x^2) \)

33. Given that \( y = \log (\log x) \) then \( \frac{dy}{dx} \) is __________
   a) \( \frac{1}{x} \) \hspace{1cm} b) \( \frac{1}{\log x} \) \hspace{1cm} c) \( \frac{1}{\log (\log x)} \) \hspace{1cm} d) \( \frac{1}{x \log x} \)

34. \( \int 2 \sin 5x \cos 2x \, dx \) is
   a) \( \sin 7x + \sin 3x \) \hspace{1cm} b) \( 7 \cos 7x + 3 \cos 3x \) \hspace{1cm} c) \( -\left( \frac{\cos 3x}{7} + \frac{\cos 3x}{3} \right) \) \hspace{1cm} d) \( \left( \frac{\cos 3x}{7} + \frac{\cos 3x}{3} \right) \)

35. \( \int \frac{x^2 + 1}{x} \, dx = \) __________
   a) \( \frac{x^3 + x}{x^2} \) \hspace{1cm} b) \( \frac{x^2}{2} + \log x \) \hspace{1cm} c) \( \frac{x^2}{2} + \frac{1}{x} \) \hspace{1cm} d) \( 1 - \frac{1}{x^2} \)

36. \( \int \sin^6 x \cos x \, dx = \) __________
   a) \( \frac{\cos^2 x}{7} \) \hspace{1cm} b) \( \sin^6 x \) \hspace{1cm} c) \( \frac{\sin^7 x}{7} \) \hspace{1cm} d) \( \sin^7 x \)

37. \( \int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 \) \( dx \) is __________
   a) \( x + \frac{1}{x} + 2 \) \hspace{1cm} b) \( \frac{x^3 + x^2}{3} \) \hspace{1cm} c) \( \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^3 \) \hspace{1cm} d) \( \frac{x^2}{2} + \log x + 2x \)

38. If \( P(A) = 0.35 \), \( P(B) = 0.73 \), \( P(A \cap B) = 0.14 \) then \( P(A \cap B) = \) __________
   a) 0.49 \hspace{1cm} b) 0.94 \hspace{1cm} c) 0.21 \hspace{1cm} d) 0.87

39. Given that \( P(A) = 0.4 \), \( P(B) = 0.5 \), \( P(A \cup B) = 0.25 \) then \( P(\overline{A} | \overline{B}) \) is
   a) 0.65 \hspace{1cm} b) 0.9 \hspace{1cm} c) 0.625 \hspace{1cm} d) 0.5

40. \( P(A) = 0.2 \), \( P(B) = 0.3 \) \( A \) and \( B \) are independent find \( P(A \cup B) = \) __________
   a) 0.44 \hspace{1cm} b) 0.6 \hspace{1cm} c) 0.5 \hspace{1cm} d) 0.15

Answer all the questions\hspace{1cm} 1 \times 40 = 40

1. The order of matrix \( B = [1 \ 2 \ 5 \ 7] \) is
   1) 1x4 \hspace{1cm} 2) 4x1 \hspace{1cm} 3) 2x1 \hspace{1cm} 4) 1x1
2. The matrix \[
\begin{bmatrix}
8 & 5 & 7 \\
0 & 6 & 4 \\
0 & 0 & 2
\end{bmatrix}
\] is
1) the upper triangular  
2) lower triangular  
3) square matrix  
4) null matrix

3. The unit vector parallel to \(\vec{i}-\vec{k}\)
1) \(\frac{\vec{i}-\vec{k}}{\sqrt{2}}\)  
2) \(\frac{\vec{i}-\vec{j}}{\sqrt{2}}\)  
3) \(\frac{\vec{i}+\vec{j}}{\sqrt{2}}\)  
4) \(\frac{\vec{i}+\vec{k}}{\sqrt{2}}\)

4. If the initial point of vector \(-2\vec{i}-3\vec{j}\) is \((-1,5,8)\) then the terminal point is
1) \(3\vec{i}+2\vec{j}+8\vec{k}\)  
2) \(-3\vec{i}+2\vec{j}+8\vec{k}\)  
3) \(-3\vec{i}-2\vec{j}-8\vec{k}\)  
4) \(3\vec{i}+2\vec{j}-8\vec{k}\)

5. If \(\frac{12x-17}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{5}{(x+1)}\) the value of \(A\) is
1) 5  
2) 7  
3) 12  
4) -17

6. If \(n! = (n-1)!\), then \(n = \)_______
1) 0  
2) 1  
3) 2  
4) 5

7. The sum of all binomial coefficients is _________
1) \(n^2\)  
2) \(n-1\)  
3) \(2^n\)  
4) \(n+1\)

8. A polygon has 44 diagonals then the number of its sides is _________
1) 11  
2) 7  
3) 8  
4) 12

9. The value of \(e^{\log x} = ?\)
1) \(x\)  
2) \(1\)  
3) \(e\)  
4) \(x > 0\)

10. The A.M., G.M. and H.M between two positive numbers \(a\) and \(b\) are equal then
1) \(a=b\)  
2) \(ab=1\)  
3) \(a>b\)  
4) \(a<b\)

11. The sum to the first 25 terms of the series \(1+2+3+\cdots\) is
a) 203  
b) 325  
c) 315  
d) 335

12. The A.M between two numbers is 5 and the G.M is 4. Then H.M. between them is
1) \(\frac{3}{5}\)  
2) 1  
3) 9  
4) \(\frac{1}{4}\)

13. \(P,Q,R\) are points on the same line with slope of \(PQ=\frac{2}{3}\), then the slope of \(QR\)
is ____________
1) \(\frac{2}{3}\)  
2) \(\sqrt{3}-\frac{2}{3}\)  
3) \(\frac{3}{2}\)  
4) \(-\frac{3}{2}\)

14. The Value of “\(m\)” for which the point \((2,3)\) lies on the line \(2x-my+11 = 0\) is
1) \(-5\)  
2) 0  
3) 3  
4) 5

15. If \(2x^2+kxy+4y^2=0\) represents a pair of parallel lines then \(k = ?\)
1) \(\pm 32\)  
2) \(\pm 2\sqrt{2}\)  
3) \(\pm 4\sqrt{2}\)  
4) \(\pm 8\)

16. \(\frac{1}{360}\) of a Complete rotation clockwise is
1) \(-1^o\)  
2) \(-360^o\)  
3) \(-90^o\)  
4) \(1^o\)

17. If \( x = a \cos^3 \theta \); \( y = b \sin^3 \theta \) then \( \left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^3 \) is equal to __________

1) \( 2 \cos^3 \theta \)  2) \( 3b \sin^3 \theta \)  3) 1  4) \( a \sin^2 \theta \cos^2 \theta \)

18. The value of \( 4 \sin 18^\circ \cos 36^\circ \) is equal to __________

1) -1  2) \( \frac{\sqrt{3}}{2} \)  3) 1  4) -\( \frac{\sqrt{3}}{2} \)

19. In any triangle \( \triangle ABC \) \( \Delta \) is

1) \( abc \)  2) \( \frac{abc}{4R} \)  3) \( \frac{abc}{2R} \)  4) \( \frac{abc}{R} \)

20. \( \tan^{-1} x + \cot^{-1} x = \) __________

1) -1  2) -\( \pi \)  3) \( \frac{\pi}{2} \)  4) \( \pi \)

21. The value of \( \frac{1 - \cos 15^\circ}{1 + \cos 15^\circ} = \) __________

1) \( \sec 30^\circ \)  2) \( \tan^2 \left( \frac{15}{2} \right) \)

3) \( \tan 30^\circ \)  4) \( \tan^2 \left( \frac{1}{2} \right) \)

22. Which of the following statements is / are true?
   a. sum of two odd functions is an odd function
   b. Inverse exists for a linear function
   c. \((2,3)\) is the superset of \([2,3]\)
   d. \(f(x) = x^2 + 1\) is not a bijective function

1) a,b  2) a,b,c  3) a,b,d  4) b,c,d

23. \( x^2 - 5x + 6 < 0 \) has ___________ solution
   a) \( x \in (2,3) \)  b) \( x \in (-\infty,2) \cup (3,\infty) \)  c) \( x \in [2,3] \)  d) None

24. Identify the correct statement
   1) The set of real numbers is a closed set
   2) The set of all – non negative real numbers is represented by \((0, \infty)\)
   3) The set \([3,7] \) indicates the set of all natural numbers between 3 and 7
   4) \((2,3)\) is a subset of \([2,3]\)

25. Which one of following is onto?
   1) \( f : \mathbb{R} \rightarrow \mathbb{R}; f(x) = x^2 \)
   2) \( f : \mathbb{R} \rightarrow \mathbb{R}; f(x) = x^2 + 1 \)
   3) \( f : \mathbb{R} \rightarrow \{1, -1\}; f(x) = x - 1 \)
   4) \( f : \mathbb{R} \rightarrow \mathbb{R}; f(x) = -x^2 \)

26. \( \lim_{x \to 1} \frac{e^x - e}{x - 1} \) is __________
   a) 1  b) 0  c) \( \infty \)  d) \( e \)

26. \( f(x) = x^3 - 8x^2 + 10 \) then \( f'(x) = \) __________
   a) 13  b) 12  c) -13  d) -12

27. \( \lim_{x \to 0} x \cot x \) is __________
   a) 0  b) -1  c) \( \infty \)  d) 1

28. \( L f'(\alpha) \) for the function \( f(x) = |x - \alpha| \) is
29. The function \( y = \tan x \) is continuous at
1) \( x = 0 \)  
2) \( x = \frac{\pi}{2} \)  
3) \( x = \frac{3\pi}{2} \)  
4) \( x = -\frac{\pi}{2} \)

30. The true statements of the following are
1. The composition of function \( f \circ g \) and the product of functions \( fg \) are same
2. For the composition of functions \( f \circ g \), the co-domain of \( g \) must be domain of \( f \)
3. If \( f \circ g \) and \( g \circ f \) exist then \( f \circ g = g \circ f \)
4. If the function \( f \) and \( g \) are having same domain and co-domain then \( fg = gf \)

1) all  
2) (ii), (iii) and (iv)  
3) (iii) and (iv)  
4) (ii) and (iv)

31. The function \( f(x) = P(A \cup B) \begin{cases} 2, & x \leq 1 \\ x, & x > 1 \end{cases} \) is not differentiable at
1) \( x = 0 \)  
2) \( x = -1 \)  
3) \( x = 1 \)  
4) \( x = -2 \)

32. \( \lim_{x \to \infty} \left( 1 - \frac{1}{x} \right)^2 \) is
1) \( e \)  
2) \(-e\)  
3) \( \frac{1}{e} \)  
4) 0

33. If \( x = ct \), \( y = \frac{c}{t} \) then \( \frac{dy}{dx} \) is
1) \( t \)  
2) \(-t \)  
3) \( \frac{1}{t^2} \)  
4) \( -\frac{1}{t^2} \)

34. \( \int \cos^2 x \, dx = ? \)
1) \( \frac{\cos^3 x}{3} + c \)  
2) \( -\frac{\sin^3 x}{3} + c \)  
3) \( \frac{1}{2} \left[ x + \frac{\sin 2x}{2} \right] + c \)  
4) \( \frac{1}{2} [1 + \sin 2x] + c \)

35. \( \int xe^x \, dx = ? \)
1) \( e^x (x - 1) + c \)  
2) \( \frac{x^2}{2} e^x + c \)  
3) \( e^x (x + 1) + c \)  
4) \( xe^x + c \)

36. \( \int \cot x \, dx = ? \)
1) \( \cosec^2 x + c \)  
2) \( \log (\sin x) + c \)  
3) \( \tan x + c \)  
4) \( \log (\cosec x) + c \)

37. \( \int \frac{3x}{x^2 + 2} \, dx = ? \)
1) \( 84 \)  
2) \( 48 \)  
3) \( 54 \)  
4) \( 18 \)

38. “The theory of probability is nothing more than good sense confirmed by calculation” said by
1) Laplace  
2) Ramanujam  
3) Newton  
4) Leibniz

39. Two coins are tossed simultaneously. What is the probability of getting exactly one head
1) \( \frac{1}{3} \)  
2) \( \frac{1}{2} \)  
3) \( \frac{3}{4} \)  
4) \( \frac{2}{3} \)

40. Given that \( P(A) = 0.35 \), \( P(B) = 0.73 \) and \( P(A \cup B) = 0.14 \) 
then the value of \( P(A \cap B) = ? \)
1) 0.94  
2) 0.49  
3) 0.59  
4) 0.21
1. The true statements of the following are
   (i) Every unit matrix is a scalar matrix
   (ii) Every scalar matrix is a diagonal matrix
   (iii) Every diagonal matrix is a square matrix
   1) all b) (i) and (ii) c) (ii) and (iii) d) (i) and (iii)

2. The cofactor of -7 in
   \[
   \begin{pmatrix}
   2 & -3 & 5 \\
   -7 & 0 & 4 \\
   6 & 1 & 5
   \end{pmatrix}
   \]
   1) 20 2) 140 3) -20 4) 12

3. If G is the centroid of a triangle ABC and O is any other point then
   \[
   \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}
   \]
   1) \(\overrightarrow{O}\) 2) \(\overrightarrow{OG}\) 3) 4 \(\overrightarrow{OG}\) 4) 3 \(\overrightarrow{OG}\)

4. \(\vec{a} = 2i + j - 8k\) ; \(\vec{b} = i + 3j - 4k\) then \(|\vec{a} + \vec{b}| = ?
   1) 13 2) 13/3 3) 3/13 4) 4/13

5. If \(\frac{12x-17}{(x-2)(x-1)} = \frac{7}{(x-2)} + \frac{B}{(x-1)}\) the value of B is
   1) 5 2) 7 3) 12 4) -17

6. If \(2x = 12\) then the value of x is
   1) 4 2) 3 3) 1 4) 2

7. In how many different ways can 4 identical toys be placed which
   form a circle
   a) 6 b) 3 c) 4 d) 24

8. The value of \(^nC_0 + ^nC_1 + ^nC_2 + \ldots + ^nC_n\) is
   1) 0 2) n 3) 2^n 4) 2^{n-1}

9. The sum to the first 25 terms of the series \(1+2+3+\ldots\) Is
   1) 203 2) 325 3) 315 4) 335

10. The A, G, H are respectively Arithmetic mean, Geometric mean and Harmonic mean
    then

12. When the terms of a G.P are written in reverse order the progression formed is
    1) A.P 3) G.P. 3) H.P. 4) A.P. and H.P

13. The angle made by the line \(x+y+7=0\) with the positive direction of x axis is
    a) 45° b) 135° c) 210° d) 60°

14. When \(ax + 3y + 5 = 0\) and \(2x + 6y + 7 = 0\) are parallel then the value of “a” is
    a) 2 b) -2 c) 1 d) 6

15. The graph of \(xy=0\) is
1) a point  
2) a line  
3) a pair of intersecting lines  
4) a pair of parallel lines

16. One radian is equal to (in terms of degree)
1) \( \frac{180^\circ}{\pi} \)  
2) \( \frac{180^\circ}{11} \)  
3) \( \frac{\pi}{180^\circ} \)  
4) \( \frac{11}{180^\circ} \)

17. The product \((s-a)(s-b)(s-c)\) is equal to
1) \( \Delta \)  
2) \( \frac{\Delta^2}{s} \)  
3) \( 2\Delta \)  
4) \( \frac{\Delta}{s} \)

18. The value of Sin \(2460^\circ\) is
1) \( \frac{1}{2} \)  
2) \( \frac{3}{4} \)  
3) \( -\frac{1}{2} \)  
4) \( \frac{3}{2} \)

19. The principle value of \(\cos x = \frac{\sqrt{3}}{2}\) is
1) \( \frac{\pi}{6} \)  
2) \( -\frac{\pi}{6} \)  
3) \( \frac{\pi}{3} \)  
4) \( -\frac{\pi}{3} \)

20. If \(\cot^{-1}\left(\frac{1}{7}\right) = \theta\) the value of \(\cos \theta\) is
1) \( 5\sqrt{2} \)  
2) \( \frac{1}{5\sqrt{2}} \)  
3) \( -5\sqrt{2} \)  
4) \( -\frac{1}{5\sqrt{2}} \)

21. The value of \(\tan 15^\circ\) is
1) \( 2 - \sqrt{3} \)  
2) \( 2 + \sqrt{3} \)  
3) \( \sqrt{3} - 2 \)  
4) \( \sqrt{3} + 2 \)

22. Which of the following is not one to one?
1) \( f: \mathbb{R} \to \mathbb{R} ; f(x) = x^2 \)  
2) \( f: \mathbb{R} \to \mathbb{R} ; f(x) = x^2 + 1 \)  
3) \( f: \mathbb{R} \to \{1, -1\} ; f(x) = \left\lfloor \frac{x}{2} \right\rfloor \)  
4) \( f: \mathbb{R} \to \mathbb{R} ; f(x) = -x^2 \)

23. If the function \(f(x) = \begin{cases} x^2 - (a + 2)x + a & \text{for } x \neq 2 \\ 2 & \text{for } x = 2 \end{cases}\) is continuous at \(x = 3\) then the value of \(a\) is
1) 3  
2) -1  
3) 0  
4) 1

24. The interval \((a, \infty)\) is called
1) finite interval  
2) infinite interval  
3) closed interval  
4) semi open interval

25. The domain of the function \(f(x) = \sqrt{1 - x^2}\) is
1) \([-1, 1]\)  
2) \((-1, 1)\)  
3) \((0, 1)\)  
4) \([0, 1]\)

26. The function \(f(x) = x^3\) is called
1) constant function  
2) identity function  
3) rational function  
4) absolute function

27. The range of the exponential function is always
1) real numbers  
2) positive real number  
3) negative real number  
4) None

28. \(\lim_{x \to 0} (1 + x)^\frac{1}{x} = ?\)
1) \(e\)  
2) \(\frac{1}{e}\)  
3) 0  
4) 1

29. Let \( f(x) = A \) and \( \overline{B} \) be the greatest integer function then
   1) \( f(x) \) is continuous at all integral values
   2) \( f(x) \) is discontinuous at all integral values
   3) \( x=0 \) is the only discontinuous point
   4) \( x=1 \) is the only continuous point

30. The function \( f(x) = \frac{x^2 + 1}{x^2 - 3x + 2} \) is continuous at all points of \( \mathbb{R} \) except at
   1) \( x = 1 \)
   2) \( x = 2 \)
   3) \( x = 1,2 \)
   4) \( x = -1, -2 \)

31. The derivative of \( f(x) = x^2 \) at \( x = 0 \) is
   1) 0
   2) -1
   3) -2
   4) 1

32. If \( f(x) = |x| + |x-1| \) is
   1) continuous at \( x = 0 \) only
   2) continuous at \( x = 1 \) only
   3) continuous at both \( x = 0 \) and \( x = 1 \)
   4) discontinuous at \( x = 0,1 \)

33. If \( x = at^2, \ y = 2at \) then \( \frac{dy}{dx} \) is
   1) \( t \)
   2) \( -t \)
   3) \( \frac{1}{t} \)
   4) \( -\frac{1}{t} \)

34. \( \int (ax+b) \ dx = \)
   1) \( \frac{(ax+b)^2}{2} + c \)
   2) \( \frac{(ax+b)^2}{2a} + c \)
   3) \( (ax+b)(x+c) \)
   4) \( \frac{ax^2}{2} + c \)

35. \( \int \frac{e^x + 1}{e^x} \ dx = ? \)
   1) \( x - e^x + c \)
   2) \( x + e^x + c \)
   3) \( \log(e^x) + c \)
   4) \( x - e^x + c \)

36. \( \int \tan^2 x \ dx = \)
   1) \( \tan x + x + c \)
   2) \( \tan x - x + c \)
   3) \( \sec^2x + c \)
   4) \( \sec x + c \)

37. \( \int \frac{1}{\sqrt{3+4x}} \ dx = \)
   1) \( \frac{1}{2} \sqrt{3+4x} + c \)
   2) \( \frac{1}{4} \log(\sqrt{3+4x}) + c \)
   3) \( 2 \sqrt{3+4x} + c \)
   4) \( -\frac{1}{2} \sqrt{3+4x} + c \)

38. If \( A \) and \( B \) are mutually exclusive events then \( P(A \cup B) = \)
   1) \( P(A) \cdot P(B) \)
   2) \( P(A) - P(B) \)
   3) \( P(A) + P(B) \)
   4) \( P(A) + P(B) - P(A \cap B) \)

39. A and B are two possible event the value of \( P\left(\frac{A}{B}\right) \) is
   1) \( \frac{P(A \cap B)}{P(A)} \)
   2) \( \frac{P(A \cap B)}{P(B)} \)
   3) \( P(A) + P(B) \)
   4) \( P(A) - P(B) \)

40. If \( A \) and \( B \) are independent event which of the following is wrong
   1) \( \overline{A} \) and \( \overline{B} \) are also independent
   3) \( A \) and \( \overline{B} \) are also independent
   2) \( \overline{A} \) and \( B \) are also independent
   4) \( A \) and \( B \) are dependent
Answer all the questions

1. If all the three rows are identical in a determinant \( \Delta \) on putting 
\( x = a \) then the factor of \( \Delta \) is
   1) \( x - a \)                     2) \( (x + a) \)     3) \( (x - a)^2 \)     4) \( (x + a)^2 \)

2. The value of 
\[
\begin{vmatrix}
1 & a & b + c \\
1 & b & c + a \\
1 & c & a + b
\end{vmatrix}
\]
is
   1) 1                     2) 0                     3) abc                     4) \( a + b + c \)

3. If \( \vec{a} \) is an non-zero vector and \( k \) is a scalar such that 
\( |k\vec{a}| = 1 \) then \( k = ? \)
   1) \( \vec{a} \)                     2) \( 1 \)                     3) \( \frac{1}{|\vec{a}|} \)     4) \( \pm \frac{1}{|\vec{a}|} \)

4. Let \( \vec{a}, \vec{b} \) be the vector \( \overrightarrow{AB}, \overrightarrow{BC} \) determined by two adjacent sides of a regular hexagon 
\( ABCDEF \). The vector represented by \( \overrightarrow{EF} \) is
   1) \( \vec{a} - \vec{b} \)                     2) \( \vec{a} + \vec{b} \)                     3) \( 2\vec{a} \)                     4) \( -\vec{b} \)

5. If 
\[
\frac{x^2 + 5}{(x^2 + 2)^2} = \frac{1}{x^2 + 2} + \frac{K}{(x^2 + 2)^2}
\]
then \( K = \) ________
   1) 1                     2) 3                     3) \( 3 \)                     4) \( 4 \)

6. In 4 places 4 persons can be seated in ________ ways
   1) \( 3! \)                     2) \( 24 \)                     3) \( 24! \)                     4) \( 5! \)

7. In how many different ways can 4 identical toys be placed which
   form a circle
   1) \( 6 \)                     2) \( 3 \)                     3) \( 4 \)                     4) \( 24 \)

8. How many different arrangements can be made out of letters of
   words ENTERTAINMENT
   a) \( \frac{13!}{3!3!3!} \)                     b) \( \frac{13!}{3!} \)                     c) \( \frac{13!}{2!2!3!} \)                     d) \( \frac{13!}{2!2!2!} \)

9. The \( 7^{th} \) term of the sequence whose \( n^{th} \) term is \((-1)^{n+1} \left( \frac{n+1}{n} \right) \)
   1) \( \frac{11}{5} \)                     2) \( -\frac{7}{8} \)                     3) \( \frac{8}{7} \)                     4) \( -\frac{8}{7} \)

10. If \( a, a+d, a+2d, a+3d \ldots \) are in A.P. the \( n^{th} \) term of H.P. is
    1) \( a + (n-1)d \)                     2) \( \frac{1}{a + (n-1)d} \)                     3) \( a + nd \)                     4) \( \frac{1}{a + nd} \)

11. The \( a_n = 2 + \frac{1}{n} \) be \( n^{th} \) term of the sequence then \( a_5 \) is
    1) \( \frac{5}{11} \)                     2) \( \frac{3}{5} \)                     3) \( \frac{10}{5} \)                     4) \( \frac{11}{5} \)

12. The arithmetic mean in between 2 and 8 is
    1) \( 10 \)                     2) \( 5 \)                     3) \( 20 \)                     4) None

13. Two lines \( ax + by + c = 0 \) and \( px + qy + r = 0 \) are perpendicular if ________
1) $\frac{a}{b} = \frac{b}{q}$  2) $\frac{a}{b} = \frac{q}{p}$  3) $\frac{a}{b} = -\frac{b}{q}$  4) $\frac{a}{b} = -\frac{q}{p}$

14. Slope of the line perpendicular to $ax + by + c = 0$ is ____________
   1) $-\frac{a}{b}$  2) $-\frac{b}{a}$  3) $\frac{b}{a}$  4) $\frac{a}{b}$

15. If the circle has both $x$ and $y$ axes as tangents and has radius 1 unit then the equation of the circle is
   1) $x^2 + (y-1)^2 = 1$  2) $x^2 + y^2 = 1$  3) $(x-1)^2 + (y-1)^2 = 1$  4) $(x-1)^2 + y^2 = 1$

16. If $\sin \theta < 0$, $\cos \theta < 0$ then $\theta$ lies in ___________
   1) 1 quadrant  2) 2 quadrant  3) 3 quadrant  4) 4 quadrant

17. $\cos 70 \cos 10 + \sin 70 \sin 10 = $ ____________
   1) $\frac{\sqrt{3}}{2}$  2) $\frac{1}{2}$  3) 1  4) $\frac{1}{\sqrt{2}}$

18. $\sin \left( A - \frac{\pi}{2} \right) = $ ______________
   1) $\sin A$  2) $-\cos A$  3) $-\sin A$  4) $\cos A$

19. $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = $ ______________
   1) $-\sin 2\theta$  2) $\cos 2\theta$  3) $\tan 2\theta$  4) $\cot 2\theta$

20. $\frac{\sin x}{x}$ of a complete rotation anticlockwise is
   1) $-1^\circ$  2) $-360^\circ$  3) $-90^\circ$  4) $1^\circ$

21. $\cos A$ is equal to
   1) $\frac{c^2 + a^2 - b^2}{2ca}$  2) $\frac{c^2 + b^2 - a^2}{2bc}$  3) $\frac{a^2 + b^2 - c^2}{2ab}$  4) $\frac{a^2 + b^2 + c^2}{2ab}$

22. If $f(x) = \frac{\sin x}{x}$, $x \neq 0$ $f(x)$ is continuous at $x = 0$ then $f(0) = $ __________
   1) 0  2) 1  3) -1  4) 2

23. The range of the function $\tan x$ is
   1) $(-\infty, \infty)$  2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  3) $[-1, 1]$  4) $[0, 1]$

24. Which of the following is not the Graph of a function
   1) $y = x^3$  2) $y = \cos x$  3) $y = e^x$  4) $x^2 + y^2 = 4$

25. The other name which is given to on to function is
   1) in to function  2) one to one function  3) surjective function  4) Bijective function

26. The Graph of the quadratic function is
   1) straight line  2) parabola  3) circle  4) pair of straight line

27. If $f: R \rightarrow R$ defined by $f(x) = x + 1$ and $g(x) = x^2$ then $gof(3)$ is equal to
   1) 16  2) 3  3) 16  4) None
28. \[ \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \]  
   1) 0  
   2) 2  
   3) 1  
   4) \( \infty \)

29. \[ \frac{5}{6} P(A \cap B) = \frac{1}{2} \]  
   1) 0  
   2) 1  
   3) 2  
   4) \( \infty \)

30. The function \( f(x) = |x| \) is ________  
   1) continuous at \( x = 0 \)  
   2) discontinuous at \( x = 0 \)  
   3) not continuous from the right at \( x = 0 \)  
   4) not continuous from the left at \( x = 0 \)

31. Given the function \( f(x) = \begin{cases} 2x - 1 & \text{if } x > 2 \\ k & \text{if } x = 2 \\ x^2 - 1 & \text{if } x < 2 \end{cases} \) is continuous then the value of \( k \) is  
   1) 2  
   2) 3  
   3) 4  
   4) -3

32. \( f: \mathbb{R} \to \mathbb{R} \) is a function defined by \( f(x) = y = 10x - 7 \) then the inverse of \( f(x) \) is  
   1) \( 10x + 7 \)  
   2) \( \frac{1}{10x - 7} \)  
   3) \( \frac{y + 7}{10} \)  
   4) \( \frac{y - 7}{10} \)

33. Given \( x^2 + y^2 = 5 \) then \( \frac{dy}{dx} \) is ________  
   1) \(-\frac{x}{y}\)  
   2) \(\frac{x}{y}\)  
   3) \(\frac{y}{x}\)  
   4) \(-\frac{y}{x}\)

34. \( y = e^{\sin x} \) then the derivative of \( y \) with respect to \( x \) is  
   1) \( y \cos x \)  
   2) \( y \sin x \)  
   3) \(-y \cos x \)  
   4) \(-y \sin x \)

35. \( \int \frac{1}{\cos^2 x} \, dx = \)  
   1) \( \tan x + c \)  
   2) \( \sec^2 x + c \)  
   3) \( \sin^2 x + c \)  
   4) \( \tan x + c \)

36. \( \int \frac{1}{1 + x^2} \, dx = \)  
   1) \( \tan^{-1} x + c \)  
   2) \( \cot^{-1} x + c \)  
   3) \( \sin^{-1} x + c \)  
   4) \( \cos^{-1} x + c \)

37. \( \int \sec x \, dx = \)  
   1) \( \log (\tan x/2) \)  
   2) \(-\log (\csc x + \cot x) + c \)  
   3) \( \log (\csc x - \cot x) + c \)  
   4) all of them

38. What is the chance that a leap year should have fifty three Mondays  
   1) \( \frac{1}{7} \)  
   2) \( \frac{2}{7} \)  
   3) 0  
   4) 1

39. If three coins are tossed, the probability of the event showing exactly one head on them is  
   1) \( \frac{1}{8} \)  
   2) \( \frac{3}{8} \)  
   3) \( \frac{5}{8} \)  
   4) \( \frac{7}{8} \)

40. A and B are two events such that \( P(A \cup B) = \frac{5}{6} \) \( P(A \cap B) = \frac{1}{3} \) and  
    \( P(B) = \frac{1}{2} \) then the event A and B are termed as  
   1) dependent  
   2) independent  
   3) mutually exclusive  
   4) impossible

TEST 5
SECTION A

Answer all the questions

1. The value of \[
\begin{vmatrix}
-a & b & c \\
a & -b & c \\
a & b & -a
\end{vmatrix}
\]
is

a) -2abc   b) a^2b^2c^2   c) 0   d) abc

2. \[\Delta = \begin{vmatrix}
a_1 & b_1 & c_1 \\
a_2 & b_2 & c_2 \\
a_3 & b_3 & c_3
\end{vmatrix}\] then \[a_1A_2, b_1B_2, c_1C_2\] is ______

a) \(\vec{a} = \vec{i} + \vec{j} - 2\vec{k}\) \(\Delta\)   b) 0   c) - \(\Delta\)   d) 1

3. The position vector of A and B are \(2\vec{i} - \vec{j} + 5\vec{k}\) and \(6\vec{i} + 3\vec{j} - 7\vec{k}\) then the position vectors of the mid point of AB is ______

a) \(6\vec{i} - \frac{3\vec{j} - 35\vec{k}}{2}\)   b) \(-3\vec{i} + 2\vec{j} - 2\vec{k}\)   c) \(8\vec{i} + 2\vec{j} - 2\vec{k}\)   d) \(4\vec{i} + \vec{j} - \vec{k}\)

4. Given \(\vec{a} = \vec{i} + \vec{j} - 2\vec{k}\) and \(\vec{b} = 3\vec{i} - \vec{j} + 3\vec{k}\) then the unit vector parallel to \(\vec{a} + \vec{b}\) is

a) \(\frac{4\vec{i} - \vec{k}}{3}\)   b) \(\frac{4\vec{i} - \vec{k}}{5}\)   c) \(\frac{4\vec{i} - \vec{k}}{17}\)   d) \(\frac{4\vec{i} - \vec{k}}{\sqrt{17}}\)

5. \[
\frac{x^2 + 4}{x^3(x + 2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 2}
\] then the value of \(C\) is

a) 4   b) 8   c) 0   d) 6

6. Ten students compute in a race. The number of ways first three prizes be given in ______

a) 720   b) 27   c) 90   d) 72

7. \((\sqrt{3} + 1)^3 - (\sqrt{3} - 1)^3\) is ______

a) 172   b) 162   c) 142   d) 152

8. There are 10 point in a plane no three of which are in the same straight line
   Excepting 4 points which are collinear. Then the number of triangles that can be formed with the vertices as their points is ______

a) 112   b) 120   c) 116   d) 124

9. Given that \(a_1 = a_2 = 2\) and \(a_n = a_{n-1} - 1\), \(n > 2\) then \(a_5\) is ______

a) 2   b) 0   c) -1   d) 1

10. \[2 \left\{ 1 + \frac{(\log 2)^2}{2!} + \frac{(\log 2)^4}{4!} + \ldots \right\}
\]

a) \(\log 2\)   b) \(e^2\)   c) \(\frac{5}{2}\)   d) infinity

11. \(\log (1+x) = x + \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\) is valid for

a) \(x > 0\)   b) for all real \(x\)   c) \(-1 < x \leq 1\)   d) \(-1 < x < 1\)

12. Given that \(\frac{5}{6}\) find cube root of \(\left(\frac{21}{25}\right) = ?\)

a) \(\frac{81}{85}\)   b) \(\frac{66}{71}\)   c) \(\frac{61}{76}\)   d) \(\frac{66}{85}\)

13. The angle between the lines 2x-3y+5=0 and 6x+4y+11=0 is ______

a) 0\(^\circ\)   b) 60\(^\circ\)   c) 45\(^\circ\)   d) 90\(^\circ\)
14. The equation \( x^2 + 3xy + 2y^2 + C \) represents a pair of line through origin then the value of \( C \) is __________
   a) 1 b) 0 c) 5 d) 6

15. The slope of the line joining the points (2,1) and (-2,-3) is __________
   a) 1 b) -3/5 c) 4 d) 0

16. If (-3,4) is a point on the terminal side of \( \theta \) then \( \sin \theta \) is __________
   a) \(-\frac{4}{5}\) b) \(-\frac{3}{5}\) c) \(-\frac{3}{4}\) d) \(\frac{4}{5}\)

17. The value of \( \sec (-1305^\circ) \) is __________
   a) \(-\frac{1}{2}\) b) \(\frac{1}{2}\) c) \(\sqrt{2}\) d) \(-\sqrt{2}\)

18. If \( \tan \theta = 3 \) then the value of \( \tan 3\theta \) is __________
   a) 9 b) 27 c) \(\frac{9}{13}\) d) \(\frac{4}{3}\)

19. \( \sin \frac{\pi}{6} \sin \frac{\pi}{3} - \cos \frac{\pi}{3} \cos \frac{\pi}{6} \) = __________
   a) infinity b) -1 c) 0 d) 1

20. If \( \cos A = \frac{3}{2} \) then the principal value of \( A \) is __________
   a) 30° b) 120° c) 150° d) 60°

21. If \( \csc \theta = -\frac{\sqrt{3}}{2} \) then the principal value of \( \theta \) is __________
   a) 30° b) 120° c) 150° d) 60°

22. Let \( A = \{1,2,3\} \), \( B = \{3,5,7,8\} \) and \( f \) from \( A \) to \( B \) is defined by
   \( f: x \rightarrow 2x+1 \) the co domain is __________
   a) \{3,5,7,8\} b) \{1,2,3\} c) \{3\} d) \{(1,3) (2,5) (3,7)\}

23. A father \( d \) has three sons \( a,b,c \). By assuming sons as a set \( A \) and father as a set \( B \)
   Then the relation “is a son of” is __________ function
   a) not b) may not be a c) identity d) constant

24. If \( f : \mathbb{R} \rightarrow \mathbb{R} \) is defined by \( f(x) = x^2 \) then the range of \( f \) is a set of all
   a) –ve whole numbers b) non negative real numbers
c) real numbers d) rational numbers

25. The principal domain for the function \( y = \tan x \) is
   a) \( \left(0, \frac{\pi}{2}\right) \) b) \((-\pi, \pi)\) c) \((0, \pi)\) d) \(\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\)

26. If \( f : \mathbb{R} \rightarrow \mathbb{R} \) is defined by \( f(x) = \frac{3x-1}{2} \) then \( f^{-1}(x) \) is __________
   a) \((2x-1)^3\) b) \((2x+1)^3\) c) \(\frac{3x-1}{2}\) d) \(\frac{2x+1}{3}\)

27. Given \( f(x) = 2x+1 \), \( g(x) = \frac{x-1}{2} \) then \( (gof)(x) \) is __________
   a) \(x\) b) \(1/x\) c) \(\frac{(2x+1)(x-1)}{2}\) d) \(\left(\frac{x-1}{2}\right)(2x+1)\)

28. \( \lim_{x \to 0} \frac{x}{|x|} \) is __________
   a) 0 b) 1 c) -1 d) does not exist

29. \( \lim_{x \to 0} \frac{5^x - 6^x}{x} \) is __________
30. Find \( R f(0) \) if \( f(x) = \frac{3x + |x|}{7x - 5|x|} \)
   a) 2 \hspace{1cm} b) 0 \hspace{1cm} c) 3/7 \hspace{1cm} d) 1/6

31. \( f(x) = \begin{cases} 2x - 1, & \text{if } x < 0 \\ 2x + 6, & \text{if } x \leq 0 \end{cases} \) find \( \lim_{x \to 0} f(x) = \)________
   a) 5 \hspace{1cm} b) 0 \hspace{1cm} c) 6 \hspace{1cm} d) -1

32. The derivative of \( e^{\tan^{-1}x} \) is \_________
   a) \( \frac{1}{e^{\tan^{-1}x}} \) \hspace{1cm} b) \( \frac{1}{\sqrt{1-x^2}} \) \hspace{1cm} c) \( e^{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}} \) \hspace{1cm} d) \( e^{\sin^{-1}x} \)

33. \( x^2 + y^2 = 1 \) then \( \frac{dy}{dx} \) is
   a) \( -\frac{y}{x} \) \hspace{1cm} b) \( -\frac{x}{y} \) \hspace{1cm} c) \( \frac{x}{y} \) \hspace{1cm} d) \( \frac{y}{x} \)

34. \( \int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx = \)________
   a) \( e^{\sqrt{x}} \cdot 2\sqrt{x} \) \hspace{1cm} b) \( \frac{1}{2} e^{\sqrt{x}} \cdot \frac{1}{2} \) \hspace{1cm} c) \( 2 e^{\sqrt{x}} \) \hspace{1cm} d) \( \frac{e^{\sqrt{x}}}{2} \)

35. \( \int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} \, dx = \)________
   a) \( x e^{\sin^{-1}x} \) \hspace{1cm} b) \( e^{\sin^{-1}x} \) \hspace{1cm} c) \( e^{\cos^{-1}x} \) \hspace{1cm} d) \( e^{\sin^{-1}x} \cdot \sin^{-1}x \)

36. \( \int \frac{dx}{x \log x} = \)________
   a) \( \frac{(x \log x)^2}{2} \) \hspace{1cm} b) \( \frac{(x \log x)^{-1}}{-1} \) \hspace{1cm} c) \( \frac{x^2}{2} \log x \) \hspace{1cm} d) \( \frac{\log(\log x)}{x} \)

37. \( \int \frac{\log x}{x} \, dx = \)________
   a) \( \frac{2}{x^2} \) \hspace{1cm} b) \( \frac{\log x}{x^2} \) \hspace{1cm} c) \( 1 \) \hspace{1cm} d) \( \frac{1}{x^2} \)

38. If \( P(A) = 0.35 \), \( P(B) = 0.73 \), \( P(A \cap B) = 0.14 \) then \( P(\overline{A \cup B}) \) is
   a) 0.59 \hspace{1cm} b) 0.94 \hspace{1cm} c) 0.54 \hspace{1cm} d) 0.86

39. Given \( P(A) = 0.4 \), \( P(B) = 0.5 \), \( P(A \cap B) = 0.25 \) then \( P(A \cup B) \) is
   a) 0.625 \hspace{1cm} b) 0.94 \hspace{1cm} c) 0.14 \hspace{1cm} d) 0.5

40. If the event \( A \) and \( B \) are independent and \( P(A) = 0.25 \), \( P(B) = 0.48 \) then \( P(A \cap B) \) is
   a) 0.23 \hspace{1cm} b) 0.12 \hspace{1cm} c) 0.78 \hspace{1cm} d) 0.73

**TEST 6**

**SECTION A**

1. In a third order determinant the cofactor of \( a_{23} \) is equal to the minor of \( a_{23} \) then the value of the minor is
   1) 1 \hspace{1cm} 2) \( \Delta \) \hspace{1cm} 3) \( -\Delta \) \hspace{1cm} 4) 0
2. If \( A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \) and \(|A| = 2\) then \(|3A|\) is

1) 54  
2) 6  
3) 27  
4) –54

3. If \( G \) is the centroid of the triangle \( ABC \), then \( \overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} \) is

1) \( \begin{pmatrix} \bar{a} + \bar{b} + \bar{c} \end{pmatrix} \)  
2) \( \overrightarrow{OG} \)  
3) 0  
4) \( \frac{1}{3} \begin{pmatrix} \bar{a} + \bar{b} + \bar{c} \end{pmatrix} \)

4. If \( \bar{a} = 2\bar{i} + \bar{j} - 8\bar{k} \) and \( \bar{b} = \bar{i} + 3\bar{j} - 4\bar{k} \) then the magnitude of \( \bar{a} + \bar{b} = ? \)

1) 13  
2) \( \frac{13}{3} \)  
3) \( \frac{3}{13} \)  
4) \( \frac{4}{13} \)

5. \( \frac{2x}{(x+1)(x^2+1)} \) can be expressed as

1) \( \frac{A}{(x+1)} + \frac{Bx+C}{x^2+1} \)  
2) \( \frac{A}{(x+1)} + \frac{B}{x^2+1} \)  
3) \( \frac{A}{x^2+1} + \frac{Bx+C}{(x+1)} \)  
4) \( \frac{A}{x^2+1} + \frac{B}{(x+1)} \)

6. The number of diagonals of a hexagon

1) 20  
2) 10  
3) 15  
4) 35

7. If \( ^nC_8 = ^nC_6 \) then the value of \( ^nC_2 \) is

1) 14  
2) 8  
3) 6  
4) 91

8. In how many ways can 7 identical beats be strung in to a ring

1) 630  
2) 7!  
3) 360  
4) 6!

9. If the \( n^{th} \) term of an A.P. is \((2n-1)\), then the sum of \( n \) terms is

1) \( n^2 - 1 \)  
2) \( 2(n-1) \)  
3) \( n^2 \)  
4) \( n^2 + 1 \)

10. When the terms of a G.P. are written in reverse order the progression formed is

1) A.P  
2) G.P  
3) H.P.  
4) A.P. and H.P.

11. The third term of a G.P. is 5, the product of its first five terms is

1) 25  
2) 625  
3) 3125  
4) 625 \( \times \) 25

12. \( e^{\log x} \) is equal to

1) \( x \)  
2) 1  
3) \( e \)  
4) \( \log e^x \)

13. Which of the following is a parallel line to \( 3x+4y+5=0 \)?

1) \( 4x+3y+6=0 \)  
2) \( 3x-4y+6=0 \)  
3) \( 4x-3y+9=0 \)  
4) \( 3x+4y+6=0 \)

14. Which of the following is the equation of a straight line that is neither parallel nor perpendicular to the straight line given by \( x+y=0 \)?

1) \( y=x \)  
2) \( y-x+2=0 \)  
3) \( 2y=4x+1 \)  
4) \( y+x+2=0 \)

15. Given that \((1,-1)\) is the center of the circle \( x^2+y^2+ax+by-9=0 \)

Its radius is

1) 3  
2) \( \sqrt{2} \)  
3) \( \sqrt{11} \)  
4) 11
16. If \( p \cot \theta = \cot 45^\circ \) then \( p \) is __________
   (i) \( \cos 45^\circ \)  (ii) \( \tan 45^\circ \)  (iii) \( \sin 45^\circ \)  (iv) \( \sin \theta \)

17. \( \sqrt{1-\cos^2 \theta} \sqrt{1-\sin^2 \theta} - \left( \frac{\cos \theta}{\csc \theta} \right) \)
   
   1) 0  2) 1  3) \( \cos^2 \theta - \sin^2 \theta \)  4) \( \sin^2 \theta - \cos^2 \theta \)

18. \( \sec^{-1} x + \csc^{-1} x = \) __________

   1) \( \frac{\pi}{2} \)  2) \( \frac{3\pi}{2} \)  3) \( -\frac{\pi}{2} \)  4) \( -\frac{3\pi}{2} \)

19. Triangle whose sides are 10 cm and 8 cm. The angle between the sides is 30°
   then the area is
   1) 20 sqcm  2) 40 sqcm  3) 30 sq.cm  4) 25 sq.cm

20. The value of \( \cos 72^\circ \) is
   1) \( P(A) < P(B) \frac{\sqrt{5} - 1}{4} \)  2) \( \frac{\sqrt{5} + 1}{4} \)  3) \( \frac{1 - \sqrt{5}}{4} \)  4) \( -\frac{1 - \sqrt{5}}{4} \)

21. The general solution of \( \sec \theta = -\sqrt{2} \) is
   1) \( \theta = 2n\pi \pm \frac{3\pi}{4} \)  2) \( \theta = 2n\pi \pm \frac{\pi}{4} \)
   3) \( \theta = n\pi + (-1)^n + \frac{\pi}{6} \)  4) \( \theta = n\pi + (-1)^n + \frac{3\pi}{4} \)

22. The domain of the function \( f(x) = \log_e x \)
   1) \( (0, \infty) \)  2) \( (-\infty, 0) \)  3) \( (-\infty, \infty) \)  4) \([0,1]\)

23. Let \( f : R \rightarrow R \) be defined by \( f(x) = 3x + 2 \) then the inverse of \( f^{-1}(x) \) is
   1) \( \frac{x-2}{3} \)  2) \( \frac{x+2}{3} \)  3) \( \frac{1}{3x+2} \)  4) None

24. The solution of \( x^2 \leq 9 \) is
   1) \((-3,3)\)  2) \([-3,3]\)  3) \((-3,3)\)  4) \([-3,3]\)

25. Sum of two odd functions is again
   1) an odd function  2) an even function  3) inverse function  4) None

26. The composition of \( f(x) = 2x+1 \) and \( g(x) = \frac{x-1}{2} \) is
   1) \( x-1 \)  2) \( x+1 \)  3) \( 1-x \)  4) \( x \)

27. Which of the following statement is wrong
   1) \( \sin x \) is an odd function
   2) \( \cos x \) is an even function
   3) \( \sin x \), \( \cos x \) and \( \tan x \) are all circular functions
   4) \( \sin x \), \( \cos x \) and \( \tan x \) are all hyperbolic function

28. \( \lim_{x \to 0} \frac{\sin 2x}{\sin 3x} \) is
   1) \( 2/3 \)  2) \( 3/2 \)  3) \( 3 \)  4) \( 2 \)

29. \( \lim_{x \to 0} \frac{5^x - 6^x}{x} = ? \)
1) \( \log \left( \frac{5}{6} \right) \)  
2) \( \log \left( \frac{6}{5} \right) \)  
3) \( \log 5 \)  
4) \( \log 6 \)

30. The left limit of \( \frac{x-2}{\sqrt{2-x}} \) is

1) 0  
2) 1  
3) 2  
4) None

31. Which function is discontinuous?

1) \( a^x, a>0 \)  
2) \( \log x, x>0 \)  
3) \( \sin x \)  
4) \( x - \lceil x \rceil \) at \( x = 3 \)

32. The right derivative (0) for the function \( f(x) = |x| \) is

1) 1  
2) 0  
3) 2  
3) -1

33. The derivative of \( \sin^{-1} x \) is

1) \( \frac{1}{\sqrt{1-x^2}} \)  
2) \( -\frac{1}{\sqrt{1-x^2}} \)  
3) \( \frac{1}{1+x^2} \)  
4) \( -\frac{1}{1+x^2} \)

34. The derivative of \( y = \tan^{-1}(e^x) \) is

1) \( \frac{e^x}{1+e^{2x}} \)  
2) \( \frac{1}{1+e^{2x}} \)  
3) \( \frac{e^{2x}}{1+e^{2x}} \)  
4) \( \frac{1}{1+e^{-x}} \)

35. Given \( x = 4t \) and \( y = \frac{4}{t} \) then the derivative of \( y \) with respect to \( x \) is

1) \( t \)  
2) \(-t \)  
3) \( \frac{1}{t} \)  
4) \( \frac{1}{t^2} \)

36. \( \int \left( \frac{\log x}{x} \right)^2 \, dx =? \)

1) \( \left( \frac{\log x}{3} \right)^2 + c \)  
2) \( \log(\log x) + c \)  
3) \( \frac{(\log x)}{3} + c \)  
4) None

37. \( \int x \cos x \, dx =? \)

1) \( x \sin x + \cos x + c \)  
2) \( x \sin x - \cos x + c \)  
3) \( x \cos x - \sin x + c \)  
4) \( x \cos x + \sin x + c \)

38. Given \( P(A) = 0.5 \), \( P(B) = 0.4 \), and \( P(A \cap B) = 0.2 \) then \( P\left( \frac{B}{A} \right) =? \)

1) 0.4  
2) 0.5  
3) 0.2  
4) None

39. Two cards are drawn one by one at random from a pack of 52 playing cards, the probability of getting two jacks if the first card is not replaced before the second card is drawn

40. If \( A \subseteq B \) then

1) \( P(A) \leq P(B) \)  
2) \( P(B) \leq P(A) \)  
3) \( P(B) < P(A) \)  
4) \( P(A) < P(B) \)

**FULL PORTION**

**SECTION B**

**TEST 1**

Answer any 10 questions \( 10 \times 6 = 60 \)

41. \( A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \)

Show that \( A^2 - 7A - 2I = 0 \)
42. Prove that 
\[
\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc
\]

43. Show that the points with position vectors \( \vec{a} - 2\vec{b} + 3\vec{c}, -2\vec{a} + 3\vec{b} - \vec{c} \) and \( 4\vec{a} - 7\vec{b} + 7\vec{c} \) are collinear.

44. Show that the vectors \( 2\vec{i} - \vec{j} + \vec{k}, 3\vec{i} - 4\vec{j} - 4\vec{k}; \vec{i} - 3\vec{j} - 5\vec{k} \) form a right angled triangle.

45. (a) If \( 5^P_r = 6^P_{r-1} \) find \( r \)
(b) If \( ^nP_4 = ^nP_6 \), find the value of \( ^{12}C_n \)

46. Prove the following by the principle of mathematical induction \( 2^{3n} - 1 \) is divisible by 7.

47. If the 5th term and 12th term of a H.P are 12 and 5 respectively, find the 15th term.

48. Show that the circle \( x^2 + y^2 - 8x - 6y + 21 = 0 \) is orthogonal to the circle \( x^2 + y^2 - 2x - 15 = 0 \).

49. Prove that 
\[
\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A
\]

50. (a) Find the derivative of \( y = \sin^{-1}(x^2 + 2x) \) with respect to \( x \)
(b) Find the derivative of \( \frac{x^2 - 1}{x^2 + 1} \) with respect to \( x \)

51. \( x = a(\theta + \sin \theta), y = a(1 - \cos \theta) \) find \( \frac{dy}{dx} \)

51. (a) \( \int \frac{1}{x^2 + 5x + 7} \) \( dx \)
(b) \( \int \frac{1}{9 + x^2} dx \)

52. Evaluate \( \int \frac{1}{x^2} \log \left( \frac{1}{x^2} \right) dx \)

53. A can hit a target 4 times in 5 shots, B 3 times in 4 shots, C 2 times in 3 shots, they fire a volley. What is the chance that the target is damaged by exactly 2 hits?

54. A coin is tossed twice. Event E and F are defined as follows: E = Head on first toss, F = head on second toss. Find (i) \( P(\frac{E}{F}) \) (ii) Are the events E and F independent?

55. If \( y = e^{\tan^{-1}x} \) Prove that \((1+x^2)y_2 + (2x-1)y_1 = 0\) 

(OR)

Prove that \( \int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) \)

TEST 2
SECTION B

Answer any 10 questions 10 x 6 = 60

41. If \( A = \begin{pmatrix} 3 & -5 \\ -4 & 2 \end{pmatrix} \) Show that \( A^2 - 5A - 14I = 0 \)
42. Prove that \[
\begin{vmatrix}
-a^2 & ab & ac \\
ab & -b^2 & bc \\
ca & bc & -c^2 \\
\end{vmatrix} = 4a^2b^2c^2
\]

43. Show that the points with position vectors \( \vec{a} - 2\vec{b} + 3\vec{c}, -2\vec{a} + 3\vec{b} + 2\vec{c} \) and 
\(-8\vec{a} + 13\vec{b} \) are collinear.

44. Prove that the points \( 2\hat{i} + 3\hat{j} + 4\hat{k}, 3\hat{i} + 4\hat{j} + 2\hat{k}, 4\hat{i} + 2\hat{j} + 3\hat{k} \) form an equilateral triangle.

45. (a) \( ^nP_4 = 20 ^nP_3 \) find \( n \)

(b) If \( ^nP_r = 5040 \) find the value of \( r \)

46. Prove the following by the principle of mathematical induction \( 5^{2n} - 1 \) is divisible by 24 for all \( n \in \mathbb{N} \)

47. Find the 4th term and 7th term of the H. P \( \frac{1}{2}, \frac{4}{13}, \frac{2}{9}, \ldots \)

48. Prove that the circles \( x^2 + y^2 - 8x + 6y - 23 = 0 \) and \( x^2 + y^2 - 2x - 5y + 16 = 0 \) are orthogonal.

49. If \( A, B, C, \) and \( D \) are angles of a cyclic quadrilateral prove that \( \cos A + \cos B + \cos C + \cos D = 0 \)

50. (a) Given \( y = \tan^{-1} \left[ \frac{\sqrt{1+x^2} - 1}{x} \right] \) Find \( \frac{dy}{dx} \)

(b) Differentiate \( xy = \tan(xy) \)

51. Find for \( x = \text{at}^2, \ y = 2\text{at} \)

52. (a) \( \int e^{x \log x^2} \, dx \)

(b) \( \int \frac{1}{\sqrt{4x^2 - 25}} \, dx \)

53. \( E \) and \( F \) are mutually exclusive and exhaustive events and \( G \) is any other event.
Prove that (i) \( G = (G \cap E) \cup (G \cap F) \) (ii) \( P(G) = P \left( \frac{G}{E} \right) P(E) + P \left( \frac{G}{F} \right) P(F) \)

54. What is the probability that (i) non leap year (ii) leap year should have fifty three Sundays

55. Prove that \( \int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \log \left( \frac{x-a}{x+a} \right) + c \)

(OR)

If \( y = (\sin^{-1} x)^2 \) Prove that \( (1-x^2) y_2 - xy_1 = 2 \)

TEST 3
SECTION B

Answer any 10 questions
41. If \( y = \log (\cos x) \), find \( y_3 \)
42. \[ \int \cos^3 x \, dx \]

43. If A and B are two independent events such that \( P(A) = 0.5 \) and \( P(A \cup B) = 0.8 \) Find \( P(B) \)

44. Evaluate \[ \int \frac{1}{x^2 - 7x + 5} \, dx \]

45. Evaluate (i) \[ \int \frac{1}{1 + 9x^2} \, dx \]
(ii) \[ \int \frac{1}{\sqrt{4x^2 - 25}} \, dx \]

46. Find the equation of the circle concentric with the circle \( x^2 + y^2 - 2x - 6y + 4 = 0 \) and having radius 7 units

47. (a) Differentiate \[ y = \cos^{-1} \sqrt{\frac{1 + \cos x}{2}} \]
(b) Find the derivative of \[ y = \log \left[ \sec x \left( \frac{\pi}{4} + \frac{x}{2} \right) \right] \]

48. If \( \cos \theta + \sin \theta = \sqrt{2} \cos \theta, \) show that \( \cos \theta - \sin \theta = \sqrt{2} \sin \theta \)

49. The first and second terms of a H.P. are \( \frac{1}{3} \) and \( \frac{1}{5} \) respectively find the 9\(^{th}\) term

50. Prove by the principle of mathematical induction
\[ 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n + 1)(2n + 1)}{6} \]
where \( n \in \mathbb{N} \)

51. (a) If \( \omega P_r = 5040 \) find the value of \( r \)
(b) Three men have 4 coats, 5 waistcoats and 6 caps. In how many ways can they wear them?

52. Prove that the points \( 2\vec{i} + 3\vec{j} + 4\vec{k}, 3\vec{i} + 4\vec{j} + 2\vec{k}, 4\vec{i} + 2\vec{j} + 3\vec{k} \) form an equilateral triangle

53. Find the unit vector parallel to \( 3\vec{a} - 2\vec{b} + 4\vec{c} \) where \( \vec{a} = 3\vec{i} - \vec{j} - 4\vec{k}, \vec{b} = -2\vec{i} + 4\vec{j} - 3\vec{k}, \vec{c} = \vec{i} + 2\vec{j} - \vec{k} \)

54. Prove that
\[ \begin{vmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{vmatrix} = (a + b + c)^3 \]

55. If \( A = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} \) Find \( k \) so that \( A^2 = kA - 2I \)

(OR)

Find the value of \( p \) if the line \( 3x + 4y - p = 0 \) is a tangent to the circle \( x^2 + y^2 = 16 \)

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**TEST 4**

**SECTION B**

Answer any 10 questions \( 6 \times 10 = 60 \)
41. (i) Find the value of \( x \) if \( \begin{bmatrix} 2x & 3 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = [0] \)

(ii) If \( A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \) and \( B = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \) are two square matrices, then show that \( |AB| = |A||B| \)

42. Show that
\[
\begin{vmatrix}
2bc - a^2 & c^2 & b^2 \\
c^2 & 2ca - b^2 & a^2 \\
b^2 & a^2 & 2ab - c^2
\end{vmatrix}
= \begin{vmatrix}
a & b & c \\
b & c & a \\
c & a & b
\end{vmatrix}
\]

43. Prove that the medians of a triangle are concurrent

44. Find the magnitude and direction cosines of the sum and difference of the vectors \( \vec{a} = \vec{i} - \vec{j} + 2\vec{k}, \vec{b} = 2\vec{i} + 3\vec{j} - 4\vec{k} \)

45. If \( n \) and \( r \) are positive integers such that \( 1 \leq r \leq n \) then \( ^nC_r = \frac{n}{r} \binom{n}{r-1} \)

46. Find the co efficient of \( x^5 \) in the expansion of \( \left( x - \frac{1}{x} \right)^{11} \)

47. If \( a \) and \( b \) are two different positive numbers then prove that A.M., G.M., H.M. are in G.P.

48. If \( A + B = 45^\circ \) Show that \( (1 + \tan A)(1 + \tan B) = 2 \) and hence deduce the value of \( \tan \left( \frac{\pi}{2} \right) \)

49. \( x = \frac{3at}{1 + t^3} \) and \( y = \frac{3at^2}{1 + t^3} \) find \( \frac{dy}{dx} \)

50. If \( A \) and \( B \) are the two points \((-2, 3)\) and \((4, -5)\) Find the equation of the locus of a point such that \( PA^2 - PB^2 = 20 \)

51. (i) Evaluate \( \int \sec^3 x \tan x \, dx \)

(ii) Evaluate \( \int x \cos 2x \, dx \)

52. Evaluate \( \int x e^{-2x} \, dx \)

53. State and prove addition theorem on probability for any two events

54. Evaluate \( \int \sec^3 x \, dx \)

55. If \( x = a(\cos \theta + \theta \sin \theta), y = a(\sin \theta - \theta \cos \theta) \) Show that \( a\theta \frac{d^2y}{dx^2} = \sec^3 \theta \)

56. TEST 5
SECTION B

Answer any 10 questions 6 x10 = 60

41. Find matrix $C$ such that if $A = \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 2 \\ 4 & -1 \end{pmatrix}$ and $5C + 2B = A$.

42. Show that $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2(x+2a)$.

43. (i) Find the sum of the vectors $\vec{i} - \vec{j} + 2\vec{k}$ and $2\vec{i} + 3\vec{j} - 4\vec{k}$ and also find the modulus of the sum. 
(ii) If $D$ is the mid point of the side BC of a triangle ABC, prove that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$.

44. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half of it.

45. $56P_{(r+6)} : 54P_{(r+3)} = 30800 : 1$ Find $r$.

46. Expand $(x^2+2y^3)^6$ using Binomial theorem.

47. Find six arithmetic mean between 3 and 17.

48. Show that the circles $x^2+y^2-2x+6y+6=0$ and $x^2+y^2-5x-6y+15=0$ touch each other.

49. Prove that $\sin A + \sin(120+A) + \sin(240+A) = 0$.

50. Given that $y=x^x$ find $\frac{dy}{dx}$.

51. (i) If $xy=100(x+y)$ find $\frac{dy}{dx}$.

(ii) $y= A \cos 4x + B \sin 4x$ where $A$ and $B$ are constants. Show that $y^2+16y=0$.

52. Integrate (i) $\int \frac{dx}{e^x + e^{-x}}$ (ii) $\int (2x-3)\sqrt{x+1} \, dx$.

53. An integer is chosen at random from the first 40 positive integers. What is the probability that the integer chosen is a prime (or) multiple of 4.

54. If $A$ and $B$ are two events such that $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\overline{B}) = \frac{1}{2}$.

Show that $A$ and $B$ are independent.

55. Differentiate $\log_{10}x$ from the first principle.

(OR)

Evaluate $\int \frac{x+1}{\sqrt{8+x-x^2}} \, dx$.

TEST 6
Section B

Answer any 10 questions $6 \times 10 = 60$
41. What is the difference between matrix and determinant?

\[
\begin{vmatrix}
y + z & x & y \\
z + x & z & x \\
x + y & y & z \\
\end{vmatrix} = (x + y + z)(x - z)^2
\]

42. Prove that

\[
\begin{align*}
\vec{i} + 3\vec{j} + \vec{k} & , 2\vec{i} - \vec{j} - \vec{k} & , 7\vec{j} + 5\vec{k}
\end{align*}
\]

are coplanar.

43. If \( \vec{a} \) and \( \vec{b} \) are Position points A and B respectively then find the Position vector of points of trisection of AB.

44. Examine whether the vectors \( \vec{i} + 3\vec{j} + \vec{k} ; 2\vec{i} - \vec{j} - \vec{k} & , 7\vec{j} + 5\vec{k} \) are coplanar.

45. Out of 18 points in a plane no three are in the same straight line except five points which are collinear. How many (i) straight lines (ii) triangles can be formed by joining them?

46. Evaluate the 7th power of 11 using Binomial theorem.

47. If b is the G.M. of a and c and x is the A.M. of a and B and y is the A.M. of b and c. Prove that

\[
\frac{a}{x} + \frac{c}{y} = 2
\]

48. Find the equation of the straight line if the perpendicular from the origin makes an angle of 120° with x-axis and the length of the perpendicular from the origin is 6 units.

49. If tan \( \alpha \) = \( \frac{1}{3} \) and tan \( \beta \) = \( \frac{1}{7} \) Show that \( 2\alpha + \beta = \frac{p}{4} \).

50. If \( y = x^3 + y^3 + a^3 + a^3 \) find \( \frac{dy}{dx} \).

51. If \( x^y = y^x \) Find \( \frac{dy}{dx} \).

52. Integrate (i) \( \int \cos 3x \sin 4x \, dx \) (ii) \( \int \frac{4x + 1}{x^5 + 3x + 1} \, dx \).

53. A problem in Mathematics is given to three students whose chance of solving are \( \frac{1}{2} \), \( \frac{1}{3} \), \( \frac{1}{4} \) respectively What is the probability that the problem is solved.

54. A can hit a target 4 times in 5 shots. B 3 times in 4 shots. C 2 times in 3 shots. They fire a volley what is the chance that the target is damaged by exactly 2 hits?

55. If \( y = \tan^{-1}\left( \frac{1 + x^2}{1 - x^2} \right) \) Show that \( \frac{dy}{dx} = \frac{2x}{1 + x^4} \).

(OR) Evaluate \( \int \frac{3x + 5}{\sqrt{x^2 - 2x + 3}} \, dx \).

**TEST 7**

**SECTION B**

Answer any 10 questions

6 x 10 = 60
41. Construct a 3x3 matrix whose elements are \((a_{ij})=2i+j\)

42. Prove that every element in a row (or) column of a determinant is multiplied by a constant ‘k’ then the value of the determinant is multiplied by k

43. Prove that the position vector of a point that divides the line joining two given points whose position vectors are \(\vec{a}\) and \(\vec{b}\) internally in the given ratio \(m:n\)

44. Show that the vectors \(5\hat{i}+6\hat{j}+7\hat{k}, 7\hat{i}−8\hat{j}+9\hat{k}, 3\hat{i}+20\hat{j}+5\hat{k}\) are coplanar

45. Resolve into partial fraction \(\frac{(x+2)}{(x+1)(x^2+1)}\)

46. Prove by Mathematical Induction \(2^n>n\) for all \(n \in N\)

47. Find the coefficient of \(x\) in the expansion of \(\frac{2x}{x^2+1}\)

48. Find the locus of P which moves such that its distance from the points (1,2) and (0,-1) are in the ratio 2:1

49. If \(\cos(\alpha−\beta) = \frac{4}{5}\) and \(\sin(\alpha−\beta) = \frac{5}{13}\) Find \(\tan 2\alpha\)

50. Find the differential coefficient of the following
(a) \(\sin^{-1}(3x-4x^3)\)  
(b) \(\frac{x^2-x+1}{x^2+x+1}\)

51. Find \(\frac{dy}{dx}\) if (a) \(x=at^2\), \(y=2at\)  
   (b) \(x=a\cos\theta\), \(y=b\sin\theta\)

52. Integrate (a) \(\int \sqrt{1+\sin 2x} \, dx\)  
   (b) \(\int \frac{x^2}{x^3+1} \, dx\)

53. A bag contains 5 white and 7 black balls 3 balls are drawn at random Find the probability that (i) all are white (ii) one white and 2 black

54. In a single throw of two dice find the probability of obtaining the (i) the sum less than 5 (ii) a sum greater than 10

55. If \(y= e^{ax} \sin bx\) Show that \(y''-2ay_1+(a^2+b^2)y=0\)

   (OR)

   Prove that \(\int \frac{dx}{\sqrt{x^2+a^2}} = \log(x+\sqrt{x^2+a^2}) + c\)
56. Prove that \[ \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \] Where \( a, b, c \) are non zero real numbers and hence evaluate the value of \[ \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} \]

57. If \( \vec{a}, \vec{b}, \text{and} \vec{c} \) are any 3 coplanar vectors then every vector \( \vec{r} \) in space can be uniquely expressed as \( \vec{r} = l\vec{a} + m\vec{b} + n\vec{c} \) for some scalar \( l, m, \text{and} n \)

58. A class contains 12 boys and 10 girls. From the class 10 students are to be chosen for a competition under the condition that at least 4 boys and at least 4 girls must be represented. The 2 girls who won the prizes last year should be included. In how many ways can the selection are made?

59. (i) Find the term independent of the expansion \( \frac{2}{x^2} + \frac{1}{x^3} \)

(ii) Evaluate \( 101^3 \) using Binomial theorem

60. If \( c \) is small compared to \( 1 \) show that \( \left( \frac{l}{l+c}\right)^\frac{1}{3} + \left( \frac{l}{l-c}\right)^\frac{1}{3} = 2 + \frac{3c^2}{4l^2} \) (approximately)

61. Find the co ordinates of the orthocenter of the triangle whose vertices are the points (-2,-1) (6,-1) and (2,5)

62. Solve \( \tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4} \)

63. If \( x \) is real, prove that \( \frac{x^2+3x-71}{x^2+2x+4} \) cannot have any value between 5 and 9

64. Differentiate \( \frac{(1-x)\sqrt{x^2+2}}{(x+3)\sqrt{x-1}} \)

65. If \( y = e^{ax} \sin bx \) prove that \( \frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0 \)

66. Evaluate \( \int e^{ax} \cos bx \ dx \)

67. Evaluate \( \int \left( \frac{dx}{\sqrt{2-3x-x^2}} \right) \) from \( x=5 \) to \( x=3 \)

68. Evaluate the definite integral as limit of sums \( \int_{x=3}^{x=5} (3x+1)dx \)

69. In a factory, machine I produces 45% of the output and Machine II produces 55% of the output. On the average 10% of the items produced by I and 5% of the items produced by II are defective. An item is drawn at random from a day’s output (i) find the probability that it is a defective item (ii) If it is defective, what is the probability that it was produced by Machine II

70. Prove that \( \sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C \) (OR)

Find the circles which cuts orthogonally each of the circles \( x^2+y^2+2x+17y+4=0 \), \( x^2+y^2+7x+6y+11=0 \) and \( x^2+y^2-x+22y+3=0 \)

TEST 2
Section C

Answer any 10 questions

10 x 10 = 100
56. Prove by factor method
\[
\begin{vmatrix}
  b+c & a-c & a-b \\
  b-c & c+a & b-a \\
  c-b & c-a & a+b
\end{vmatrix} = 8abc
\]

57. If \( \vec{a} \) and \( \vec{b} \) are the vectors determined by two adjacent sides of a regular hexagon find the vectors determined by the other sides taken in order.

58. (i) If \( ^{15}C_r : ^{15}C_{r-1} = 11 : 5 \), find \( r \)
(ii) \( 2^nC_3 = \frac{20}{3} nC_2 \)

59. Prove by induction method that \( 7^{2n} + 16n - 1 \) is divisible by 64

60. In the expansion of \((1+x)^{20}\), the coefficient of \( r^{th} \) and \((r+1)^{th}\) terms are in the ratio 1:6 find the value of \( r \)

61. (i) For what value of \( m \) the three straight lines \( 3x+y+2=0 \), \( 2x-y+3=0 \) and \( x+my-3=0 \) are concurrent?
(ii) Find the values of \( p \) for which the straight lines \( 8px+(2-3p)y+1=0 \) and \( px+8y-7=0 \) are perpendicular to each other

62. Solve the triangle for given three sides \( a=8 \), \( b=9 \), \( c=10 \)

63. If \( x \) is real, prove that \( \frac{\pi}{x^2 - 5x + 9} \) is lies between \( -\frac{1}{11} \) and 1

64. Find \( \frac{dy}{dx} \) if \( x^m y^n = (x+y)^{m+n} \)

65. If \( y = \sin(ax + b) \) Prove that \( y_3 = a^3 \sin \left( \frac{ax+b+3\pi}{2} \right) \)

66. Evaluate : \( \int x \sin^{-1}xdx \)

67. Evaluate : \( \int \frac{2x + 3}{x^2 - 2x + 5} dx \)

68. A consulting firm rents car from three agencies such that 20% from agency X, 30% from agency Y and 50% from agency Z. If 90% of the cars from X, 80% of cars from Y and 95% of the cars from Z are in good conditions (i) What is the probability that the firm will get a car in good condition? Also (ii) If a car is in good condition, what is probability that it has came from agency Y?

69. If \( A+B+C = \pi \)
(i) Prove that \( \tan A + \tan B + \tan C = \tan A \tan B \tan C \)
(ii) Prove that \( \tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C \)

70. Show that each of the circles \( x^2+y^2+4y-1=0 \), \( x^2+y^2+6x+y+8=0 \) and \( x^2+y^2-4x-4y-37=0 \) touches the other two

**TEST 3**

**SECTION C**

Answer any 10 questions

| 10 x 10 = 100 |
57. Prove that \( \frac{\sin^2 A}{2} + \frac{\sin^2 B}{2} + \frac{\sin^2 C}{2} = 1 - 2\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \)

58. A factory has two machines I and II. Machine I and II produces 30% and 40% of items respectively. Further 3% of items produced by Machine I are defective and 40% of items produced by Machine II are defective. An item is drawn at random. Find the probability that it was produced by Machine II.

59. Evaluate the definite integral as limit of sums \( \int_{a}^{b} (2x + 3) \, dx \)

60. Prove that \( \int \sqrt{x^3 + a^3} \, dx = \frac{x}{2} \sqrt{x^3 + a^3} + \frac{a^2}{2} \left[ \log(x + \sqrt{x^3 + a^3}) + c \right] \)

61. If \( x = \sin t \); \( y = \sin pt \) Show that \( (1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0 \)

62. If \( x \in \mathbb{R} \), Prove that the range of the function \( f(x) = \frac{x^2 - 3x + 4}{x^2 + 3x + 4} \) is \( \left[ 1, 7 \right] \)

63. State and prove any one Napier’s formula.

64. Find the equation of the straight line which passes through the intersection of the straight lines \( 5x - 6y = 1 \) and \( 3x + 2y + 5 = 0 \) and is perpendicular to the straight line \( 3x - 5y + 11 = 0 \).

65. Show that the points given by the vectors \( 4i + 5j + k, -j - k, 3i + 9j + 4k \) and \( -4i + 4j + 4k \) are coplanar.

66. Prove by vector method that the internal bisectors of the angles of a triangle are concurrent.

67. Resolve into partial fraction \( \frac{7x^2 - 25x + 6}{(x^2 - 2x - 1)(3x^2 - 2)} \)

68. If \( a, b, c \) are in A.P. and \( a, mb, c \) are in G.P. then prove that \( a, mb, c \) are in H.P.

69. In how many ways player for a cricket team can be selected from a group of 25 players containing 10 batsman, 8 bowlers, 5 all rounder and 2 wicket keepers? Assume that the team requires 5 batsman, 3 all rounder, 2 bowlers and 1 wicket keeper.

(Or)

70. Prove by factor method \( \frac{\sqrt[3]{a}}{\sqrt[3]{b}} \cdot \frac{\sqrt[3]{a}}{\sqrt[3]{c}} = (a - b)(b - c)(c - a)(ab + bc + ca) \)

TEST 4
SECTION C

Answer any 10 questions \( 10 \times 10 = 100 \)

56. If \( x, y, z \) are all different and \( \begin{vmatrix} x & x^2 & 1-x^3 \\ y & y^2 & 1-y^3 \\ z & z^2 & 1-z^3 \end{vmatrix} = 0 \) then Show that \( xyz = 1 \)

57. By using vectors the mid points of two opposite sides of a quadrilateral and the mid points of the diagonals are the vertices of a parallelogram.

58. A class contains 12 boys and 10 girls. From the class 10 students are to be chosen for a competition under the condition that at least 4 boys and at least 4 girls must be represented. The 2 girls who won the prizes last year should be included. In how many ways can the selection are made?

59. Prove the principle of mathematical induction, The sum \( S_n = n^3 + 3n^2 + 5n + 3 \) is divisible by 3 for all \( n \in \mathbb{N} \).

60. If the coefficient of 5th, 6th, and 7th terms in the expansion of \((1+x)^n\) are in Arithmetic progression find \( n \).

61. (i) If \( ax + by + c = 0 \), \( bx + cy + a = 0 \) and \( cx + ay +b = 0 \) are concurrent, show that \( a^3 + b^3 + c^3 = 3abc \).
(ii) Find the equation of the straight line joining the point (4, -3) and the intersection of the lines \( 2x - y + 7 = 0 \) and \( x + y - 1 = 0 \).

62. Prove that \( \frac{\sin 300^0 \tan 330^0 \sec (-420^0)}{\cos 135^0 \cos 210^0 \csc 315^0} = \frac{2}{3} \).

63. A function \( f(x) \) is defined by \( f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ 5x - 4 & \text{if } 0 < x \leq 1 \\ 3x + 4 & \text{if } x \geq 2 \\ 4x^2 - 3x & \text{if } 1 < x < 2 \end{cases} \).

Examine the continuity of the function at \( x = 0, 1, 2 \).

64. Given \( y = \cot^{-1} \left[ \frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}} \right] \), find \( \frac{dy}{dx} \).

65. If \( y = \sin \left( m \sin^{-1} x \right) \) Show that \((1-x^2)y_2 - xy_1 + m^2 y = 0\).

66. Evaluate \( \int \frac{4x - 3}{\sqrt{x^2 + 2x - 1}} \, dx \).

67. Prove that \( \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + c \).

68. A problem in Mathematics is given to three students whose chances of solving it are \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \).

(i) what is the probability that the problem is solved (ii) what is the probability that exactly one of them will solve it.

69. Show that \( 3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13 \).

70. Find the equation of the circle which passes through the point (1, 2) and cut orthogonally each other of the circles \( x^2 + y^2 = 9 \) and \( x^2 + y^2 - 2x + 8y - 7 = 0 \).

TEST 5

SECTION C

Answer any 10 questions 10 x 10 = 100

56. Prove by factor method
\[
\begin{vmatrix}
1 & a^2 & a^3 \\
1 & b^2 & b^3 \\
1 & c^2 & c^3 \\
\end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)
\]

57. Prove by vector method that the internal bisector of the angles of a triangle are concurrent.
58. (i) Prove that \( ^nC_{r-1} + ^nC_r = ^{n+1}C_r \)

(ii) Find the number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated

59. Prove by Induction Method \[ \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n} \]

60. (i) Show that the middle term in the expansion of \((1+x)^{2n}\) is

\[ 1.3.5.7\ldots\frac{(2n-1)2^n x^n}{n!} \]

(ii) Find the constant term in the expansion of \(\frac{\text{e}^x}{x^0} - \frac{2}{x^0} \frac{d^0}{x^0} \)

61. Show that the equation \(3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0\) represents a pair of straight lines and also find the angle between the straight lines

62. Find the equation of the circle which passes through \((1,-1)\) and cuts orthogonally each of the circles \(x^2 + y^2 + 5x - 5y + 9 = 0\) and \(x^2 + y^2 - 2x + 3y - 7 = 0\)

63. Discuss continuity of the function \( f(x) = |x-1| + |x-2| \) at \( x = 1 \) and \( x = 2 \)

64. Find the value of \( \sin 18^\circ \) and hence deduce the value of \( \cos 36^\circ \)

65. (i) Differentiate \( y = \tan^{-1} \left( \frac{\cos x + \sin x}{\cos x - \sin x} \right) \)

(ii) Find \( \frac{dy}{dx} \) when \( x = \cos^3 t \), \( y = \sin^3 t \)

66. If \( y = \log(x^2 - a^2) \) find \( \frac{d^3 y}{dx^3} = 2 \left[ \frac{1}{(x+a)^2} + \frac{1}{(x-a)^2} \right] \)

67. Evaluate \( \int_2^x \frac{3x+2}{x^2 + x + 1} \, dx \)

68. Evaluate the using limit and summation \( \int_1^x x^2 \, dx \)

69. There are two identical boxes containing respectively 5 white and 3 red balls, 4 white and 6 red balls. A box is chosen at random and a ball is drawn from it (i) find the probability that the ball is white (ii) if the ball is white, what is the probability that it is from the first box.

70. Prove that \( \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16} \)

**TEST 6**

**SECTION C**

Answer any 10 questions

<table>
<thead>
<tr>
<th>10 x 10 =100</th>
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</thead>
</table>
| 56. Prove that | \[
\begin{bmatrix}
     b+c & c+a & a+b \\
     q+r & r+p & p+q \\
y+z & z+x & x+y \\
\end{bmatrix} = \begin{bmatrix}
     a & b & c \\
p & q & r \\
x & y & z \\
\end{bmatrix}
\]

57. Show that the points given by the vectors \(4\vec{i} + 5\vec{j} + \vec{k}, -\vec{j} - \vec{k}, 3\vec{i} + 9\vec{j} + 4\vec{k}\) and \(-4\vec{i} + 4\vec{j} + 4\vec{k}\) are coplanar
58. (i) Find the number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated.
(ii) Prove that \( nP_r = (n-1)P_r + r(n-1)P_{r-1} \)

59. The first 3 terms in the expansion of \((1-ax)^n\) are \(1-14x+84x^2\) find \(a\) and \(n\)

60. If \(a, b, c\) are in G.P. Prove that \(\log_m a\), \(\log_m b\) and \(\log_m c\) are in H.P.

61. Show that the equation \(4x^2+4xy+y^2-6x-3y-4=0\) represents a pair of parallel lines and find the distance between them.

62. If \(\sin \theta = \frac{11}{12}\), find the value of
\[\sec(360^\circ - \theta)\tan(180^\circ - \theta) + \cot(90^\circ + \theta)\sin(270^\circ + \theta)\]

63. Given that \(f(x) = 2x+1\), \(g(x) = x^2+2\) and \(h(x) = x^2-1\) Show that \(f \circ (g \circ h) = f \circ g \circ h\)

64. Differentiate \(y = \log_e (2x+3)\) from first principle.

65. Differentiate \(y = \tan^{-1}\left(\frac{\cos 2x + \sin 2x}{\cos 2x - \sin 2x}\right)\)

66. Evaluate \(\int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx\)

67. Evaluate \(\int \frac{dx}{1 + \sin x + \cos x}\)

68. Evaluate the definite integral as limit of Sum \(\int_1^3 e^x dx\)

69. X speaks truth in 95% of cases and Y in 90% of cases. In what percentage of cases they are likely to contradict each other in stating the same fact?

70. Find the equation of the circle whose centre is on the line \(x=2y\) and which passes through the point \((-1,2)\) and \((3,2)\)

**TEST 7**

**Section C**

Answer any 10 questions \(10 \times 10 = 100\)

56. Show that
\[
\begin{vmatrix}
1 & x & x^2 \\
1 & y & y^2 \\
1 & z & z^2
\end{vmatrix} = (a - x)^2 (b - x)^2 (c - x)^2 - (a - y)^2 (b - y)^2 (c - y)^2 - (a - z)^2 (b - z)^2 (c - z)^2
\]

57. If \(\vec{a}\) and \(\vec{b}\) are any two vectors and \(m\) is any scalar then prove that \(m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}\)

58. (i) How many numbers can be formed with the digits 1,2,3,4,3,2,1 so that the odd digits always occupy the odd places?
(ii) How many arrangements can be made with the letters of the word MATHEMATICS?

59. If the coefficient of 5th, 6th, and 7th terms in the expansion of (1+x)^n are in Arithmetic progression find n.

60. If x is so large prove that \( \sqrt{x^2 + 25} - \sqrt{x^2 + 9} = \frac{8}{x} \) nearly.

61. Show that the equation 3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0 represents a pair of straight lines and also find the angle between the straight lines.

62. If A + B = 45° Show that ( Cot A - 1)( Cot B - 1) = 2 and deduce the value of Cot \( \frac{22}{2} \).

63. When x is real show that \( \frac{x^2 - 3x + 3}{x^2 - 3x + 2} \) cannot have any value between -3 and 1.

64. Differentiate \( \tan^{-1} \left( \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) \) with respect to x.

65. Differentiate \( y = e^{2x^3} \) from the first principle.

66. Integrate \( \int x \sin^2 x \, dx \)

67. Evaluate \( \int (x + 4)\sqrt{2x + 3} \, dx \)

68. Evaluate the definite integral as the limit of Sum \( \int_{x=1}^{x=2} (4x + 1) \, dx \)

69. A problem is given to 3 students X, Y, and Z whose chances of solving it are \( \frac{1}{2}, \frac{1}{3}, \) and \( \frac{2}{5} \) respectively. What is the probability that the problem is solved?

70. Find the equation of the tangent to the circle \( x^2 + y^2 = 9 \) which are parallel to \( 2x + y - 3 = 0 \) (OR) Prove that \( \sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - 2\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \).