I

1) 4 2) 1 3) 3 4) 4 5) 1 6) 1 7) 3 8) 3 9) 3 10) 3
11) 2 12) 3 13) 2 14) 3 15) 1 16) 3 17) 3 18) 3 19) 3 20) 4

II

21) $\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

$= \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$

$\Delta = 0$

22) $\vec{DB} = \vec{OB} - \vec{OD}$

$= 3\vec{b} - \vec{a}$

$\vec{AC} = \vec{OC} - \vec{OA}$

$= \vec{a} + 3\vec{b}$

23) $|2\vec{a} - \vec{b} + 3\vec{c}| = |11\vec{i} - 8\vec{k}|$

$= \sqrt{185}$ units

24) $R.P = \dfrac{5!}{2!\,1!\,1!\,0!\,1!} = 60$

25) $nC_{10} = nC_{12}$

$nC_{n-10} = nC_{12}$

$n = 22$

$23C_n = 23C_{22} = 23$

26) $a_5 = 2 + \frac{1}{5}$

$= \frac{11}{5}$

$a_7 = 2 + \frac{1}{5}$

$= \frac{15}{7}$

27) let $1, x_1, x_2, x_3, x_4, x_5, 19$ be in A.P.

Common difference $d = 3$

$x_1 = 1+3 = 4$

$x_2 = 4+3 = 7$

$x_3 = 7+3 = 10$

$x_4 = 10+3 = 13$

$x_5 = 13+3 = 16$

28) $2x + y - 9 = 0$

$M_1 = -2$

$2x + y - 10 = 0$

$M_2 = -2$

$M_1 = M_2$

29) $\sqrt{(x_1-1)^2 + (y_1+4)^2} = 6$

$(x_1-1)^2 + (y_1+4)^2 = 36$

locus of (x_1, y_1) is $x^2 + y^2 - 2x + 8y - 19 = 0$

30) $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} 0+c^2+b^2 & 0+0+ab & 0+ac+0 \\ 0+0+ab & c^2+0+a^2 & bc+0+0 \\ 0+ac+0 & bc+0+0 & b^2+a^2+0 \end{vmatrix}$

$= \begin{vmatrix} c^2+b^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & b^2+a^2 \end{vmatrix}$

III

31) $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

$A^2 = \begin{bmatrix} 16 & 21 \\ 28 & 37 \end{bmatrix}$

$-7A = \begin{bmatrix} -14 & -21 \\ -28 & -35 \end{bmatrix}$

$-2I = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

$A^2 - 7A - 2I = \begin{bmatrix} 16-14-2 & 21-21+0 \\ 28-28+0 & 37-35-2 \end{bmatrix}$

$= 0$

32) $\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & c^2 \end{vmatrix}$

$\Delta = abc \begin{vmatrix} -a & b & c \\ b & -b & c \\ c & b & -c \end{vmatrix}$

$\Delta = a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix}$ $R_2 \to R_2 + R_1$, $R_3 \to R_3 + R_1$

$= 4a^2b^2c^2$

33) $\vec{AB} = \vec{i} - 3\vec{j} - 5\vec{k}$

$|\vec{AB}| = \sqrt{35}$

$\vec{BC} = -2\vec{i} + \vec{j} - \vec{k}$

$|\vec{BC}| = \sqrt{6}$

$\vec{CA} = \vec{i} + 2\vec{j} + 6\vec{k}$

$|\vec{CA}| = \sqrt{41}$

$|\vec{CA}|^2 = |\vec{AB}|^2 + |\vec{BC}|^2$

$\therefore \Delta ABC$ is right angle

34) No. Selection

$= 6C_3 \times 5C_2$

$= \frac{6\times5\times4}{1\times2\times3} \times \frac{5\times4}{1\times2}$

$= 200$

35) $\left(x + \frac{1}{x^3}\right)^{17}$

$T_{r+1} = 17C_r\, x^{17-4r}$

Let T_{r+1} be the term containing x^5

$17 - 4r = 5$

$r = 3$

$T_{r+1} = T_{3+1}$

$= 17C_3\, x^{17-4(3)}$

$= 680x^5$

$\therefore$ Co-efficient of $x^5 = 680$

36) $\sum_{n=1}^{\infty} \frac{1}{3^n}$

$S_n = \frac{1}{3} + \frac{1}{3^2} + \dots + \frac{1}{3^n} + \dots$

$S_{n+1} = \frac{1}{3} + \frac{1}{3^2} + \dots + \frac{1}{3^n} + \frac{1}{3^{n+1}}$

$S_{n+1} = S_n + \frac{1}{3^{n+1}}$

$= \frac{1}{3}[1 + S_n]$

$S_n + \frac{1}{3^{n+1}} = \frac{1}{3} + \frac{1}{3} S_n$

$S_n = \frac{1}{2}\left[1 - \frac{1}{3^n}\right]$

37) Let G_1, G_2, G_3, G_4, G_5 be 5 G.M between 576 & 9

$r = 1/2$

$G_1 = 288$, $G_2 = 144$

$G_3 = 72$, $G_4 = 36$

$G_5 = 18$

38) length $= \frac{2(2)-(-3)+9}{\sqrt{(2)^2+(-1)^2}}$

$= \frac{16}{\sqrt{5}}$ units

39) Let (x_1, y_1) intersection of

$3x_1 + 4y_1 = 13 \to ①$

$2x_1 - 7y_1 = -1 \to ②$

$x_1 = 3, y_1 = 1$

Pt $(3, 1)$

Sub Pt:

$5x - y = 14$

L.H.S:-

$= 5(3) - 1$

$= 15 - 1$

$= 14$

40) $\vec{AB} = -3\vec{a} + 5\vec{b} - 4\vec{c}$

$\vec{BC} = 6\vec{a} - 10\vec{b} + 4\vec{c}$

$\vec{BC} = -2(\vec{AB})$

A, B, C are collinear

IV

41) $2x + y = \begin{bmatrix} 2 & -1 & -3 \\ -5 & 7 & -3 \\ -4 & -5 & -4 \end{bmatrix} \to ①$

$x - y = \begin{bmatrix} 4 & 7 & 0 \\ -1 & 2 & -6 \\ -2 & 8 & -5 \end{bmatrix} \to ②$

$① + ②$ $3x = \begin{bmatrix} 6 & 6 & -3 \\ -6 & 9 & -9 \\ -6 & 3 & -9 \end{bmatrix}$

$x = \begin{bmatrix} 2 & 2 & -1 \\ -2 & 3 & -3 \\ -2 & 1 & -3 \end{bmatrix}$

Sub x in ① or ②

$y = \begin{bmatrix} -2 & -5 & -1 \\ -1 & 1 & 3 \\ 0 & -7 & 2 \end{bmatrix}$

(or)

$\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

$= \begin{vmatrix} a & -b & 0 \\ 0 & b & -c \\ 1 & 1 & 1+c \end{vmatrix}$ $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$= a[b(1+c) + c] + b[0 + c$

$= abc\left[1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right]$

$\therefore \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} =$

$= a^3\left(1 + \frac{3}{a}\right)$

$= a^2(3 + a)$

42) $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$

Put $a=0$

$\Delta = \begin{vmatrix} (b+c)^2 & 0 & 0 \\ b^2 & c^2 & b^2 \\ c^2 & c^2 & b^2 \end{vmatrix}$

$\Delta = 0$

$\therefore (a-0) = a$ is factor of Δ

Illy b, c are also factor of Δ

Put $a+b+c=0$

$\Delta = \begin{vmatrix} (-a)^2 & a^2 & a^2 \\ b^2 & (-b)^2 & b^2 \\ c^2 & c^2 & (-c)^2 \end{vmatrix} = 0$

The other factor must be $k(a+b+c)$

$\therefore \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = k\,abc\,(a+b+c)^3$

Take $a=1$ $b=1$ $c=1$

$27k = 54$

$k = 2$

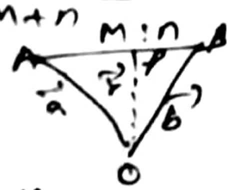
$\therefore \Delta = 2abc(a+b+c)^3$

(or)

State:

The position vector of a point that divides the line joining two points whose position vectors are $\vec{a}$ & $\vec{b}$ internally in the given ratio $m:n$ is $\dfrac{m\vec{b}+n\vec{a}}{m+n}$

proof)



$\dfrac{\vec{AC}}{\vec{CB}} = \dfrac{m}{n}$

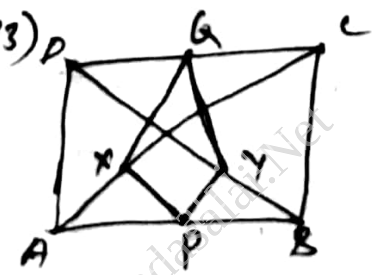
$n\vec{oc} - n\vec{oa} = m\vec{oB} - m\vec{oc}$

$(m+n)\vec{oc} = m\vec{b} + n\vec{a}$

$\therefore \vec{oc} = \dfrac{m\vec{b}+n\vec{a}}{m+n}$

$\vec{r} = \dfrac{m\vec{b}+n\vec{a}}{m+n}$

43)



$\vec{OP} = \dfrac{\vec{OA}+\vec{OB}}{2}$

$\vec{Ox} = \dfrac{\vec{OA}+\vec{OC}}{2}$

$\vec{OQ} = \dfrac{\vec{OC}+\vec{OD}}{2}$

$\vec{Oy} = \dfrac{\vec{OB}+\vec{OD}}{2}$

$\vec{Py} = \dfrac{\vec{d}-\vec{a}}{2}$

$\vec{xQ} = \dfrac{\vec{d}-\vec{a}}{2}$

$\therefore \vec{Py} \parallel \vec{xQ}$

Illy $\vec{Px} \parallel \vec{yQ}$

Hence $PyQx$ is a Parallogram

(or)

$\vec{AB} = -4\vec{i} - 6\vec{j} - 2\vec{k}$

$\vec{B} = 3\vec{i} + 10\vec{j} + 5\vec{k}$

$\vec{CD} = 7\vec{i} - 5\vec{j}$

Let

$-4\vec{i} - 6\vec{j} - 2\vec{k} = x(3\vec{i} + 10\vec{j} + 5\vec{k}) + y(-7\vec{i} - 5\vec{j})$

$x = -2/5$

$y = 2/5$

Here $a = bx + cy$

$\therefore$ coplanar.

44) $\frac{x+1}{(x-2)^2(x+3)} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$x+1 = A(x-2)^2 + B(x-2)(x+3) + C(x+3)$

Put $x=-3$ $A = -2/25$

Put $x=2$ $C = 3/5$

Equate the coefficient of x^2

$B = 2/25$

P.F $\frac{-2/25}{x+3} + \frac{2/25}{x-2} + \frac{3/5}{(x-2)^2}$

(or)

$P(n) \Rightarrow 1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

Put $n=1$

$P(1) = 1$

P(1) is true.

$P(k) \Rightarrow 1^2+2^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$

To prove

P(k+1) is true.

$1^2+2^2+\cdots+k^2+(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

$\therefore P(k+1)$ is true

Thus P(k) is true

$1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} \in N$

45) $T_{r+1} = nC_r\, x^{n-r} a^r$

$T_r : T_{r+1} = 1:6$

$\frac{20C_{r-1}}{20C_r} = \frac{1}{6}$

$\frac{20!}{r(r-1)!(20-r)!} = 6\, \frac{20!}{(r-1)!(21-r)(20-r)!}$

$\frac{1}{r} = \frac{6}{21-r}$

$\boxed{r=3}$

(or)

H.P be $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}+\cdots$

$a=3$

$d=2$

9^{th} term $= \frac{1}{a+8d}$

$= \frac{1}{3+16}$

$= \frac{1}{19}$

46) (i) A.M, G.M, H.M are in G.P

$A.M = \frac{a+b}{2}$

$G.M = \sqrt{ab}$

$H.M = \frac{2ab}{a+b}$

$\frac{G.M}{A.M} = \frac{2\sqrt{ab}}{a+b} \rightarrow ①$

$\frac{H.M}{G.M} = \frac{2\sqrt{ab}}{a+b} \rightarrow ②$

① & ②

$\frac{G.M}{A.M} = \frac{H.M}{G.M}$

∴ AM, GM, HM are in G.P

ii) $A.M > G.M > H.M$

$A.M - G.M = \frac{(\sqrt{a}-\sqrt{b})^2}{2} > 0$

$AM > G.M \rightarrow ①$

$G.M - H.M = \frac{\sqrt{ab}(\sqrt{a}-\sqrt{b})^2}{a+b} > 0$

$\therefore GM > H.M \rightarrow ②$

from ① & ②

$A.M > G.M > H.M$

(or)

$(x^3+6)^{1/3} - (x^3+3)^{1/3}$

$= x\left(1+\frac{6}{x^3}\right)^{1/3} - x\left(1+\frac{3}{x^3}\right)^{1/3}$

$= \left[x + \frac{2}{x^2} + \cdots\right] - \left[x + \frac{1}{x^2} + \cdots\right]$

$= \frac{2}{x^2} - \frac{1}{x^2} + \cdots$

$= 1/x^2$ (APP)

47)

$4x-3y-18=0 \quad m_1 = \frac{4}{3}$

$3x-4y+16=0 \quad m_2 = \frac{3}{4}$

$x+y-2=0 \quad m_3 = -1$

$\theta = \tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

angle $4x-3y-18=0$ &

$3x-4y+16=0$

$\alpha = \tan^{-1}\left(\frac{7}{24}\right)$

angle $3x-4y+16=0$

$x+y-2=0$

$\beta = \tan^{-1}(7)$

angle $x+y-2=0$

$4x-3y-18=0$

$\gamma = \tan^{-1}(7)$

$\beta = \gamma$

∴ The Δ is isosceles.

(or)

$3x^2+10xy+8y^2+14x+22y+15=0$

$a=3 \; h=5 \; b=8 \; g=7 \; f=11 \; c=15$

[redacted]

cond

$af^2+bg^2+ch^2-abc-2fgh$

$=3(11)^2+8(7)^2+15(5)^2-3(8)(15)$

$-2(11)(7)(5)$

$=363+392+375-360-770$

$=0$

$\tan\theta = \frac{\pm 2\sqrt{h^2-ab}}{a+b}$

$= \frac{\pm 2\sqrt{25-3(8)}}{3+8}$

$= \pm \frac{2}{11}$

$\tan\theta = \frac{2}{11}$

$\theta = \tan^{-1}\left(\frac{2}{11}\right)$

By

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[redacted]